Productivity Measurement and Analysis: Techniques and Applications

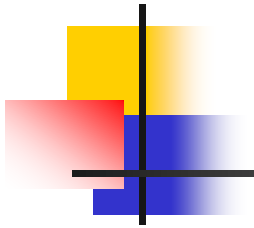
Bishwanath Goldar

Institute of Economic Growth, Delhi

November 2015

Outline of Presentation

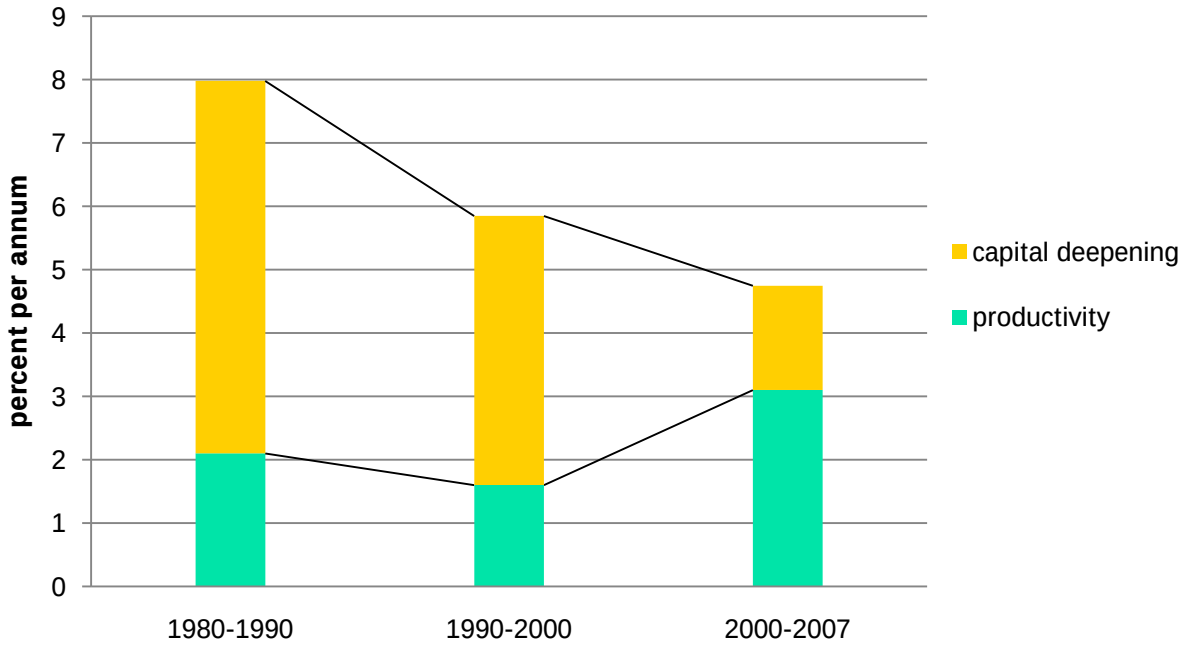
- Introduction: insights gained from productivity analysis, two examples
- Alternate approaches to productivity measurement
- Conceptual framework
- Alternate Methodology and applications
  - Total factor productivity (TFP) Indices (Tornqvist)
  - Multilateral TFP Index
  - Data Envelopment Analysis (DEA) – technical efficiency
  - Malmquist Index – technical change and technical efficiency change
  - Levinsohn-Petrin Methodology – TFP estimates
  - Stochastic frontier production function (not discussed).



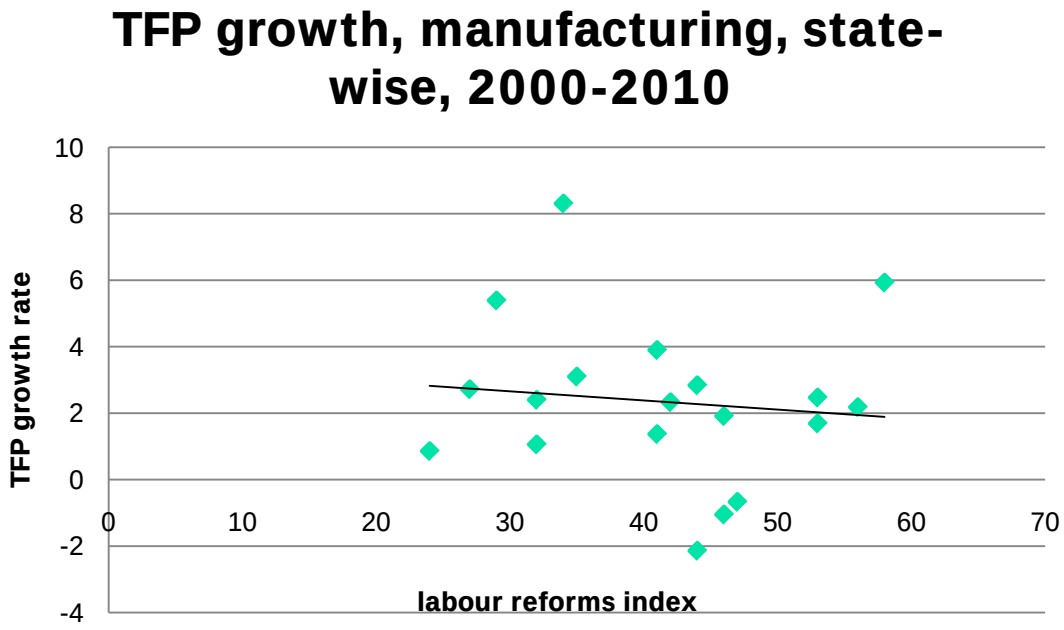
Insights gained from productivity analysis, two examples relating to labour

Growth rate in labour productivity manufacturing has slowed down in the 2000s, but labour productivity gains are now arising mostly from hikes in TFP, not capital deepening

Sources of labour productivity growth, Indian manufacturing (organized)

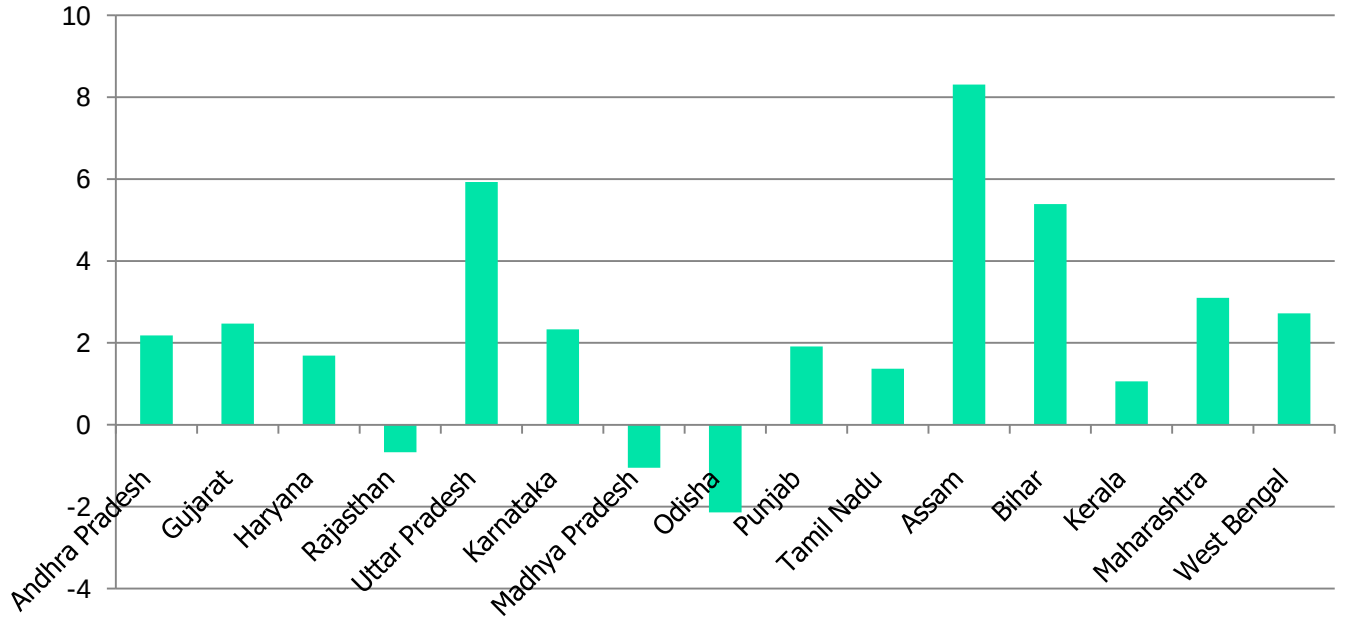


TFP is the ratio of output to total input. It represents the overall productivity or efficiency in input use. The main source of TFP increase is technical change. However, the methods commonly used tend to include economies of scale and technical efficiency gains



State that have been ahead of others in labour reforms have not achieved a faster growth rate in TFP

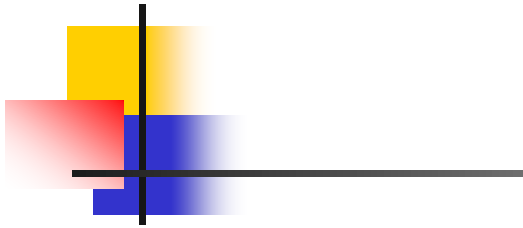
TFP growth rate, manufacturing, state-wise, 2000-2010



flexible

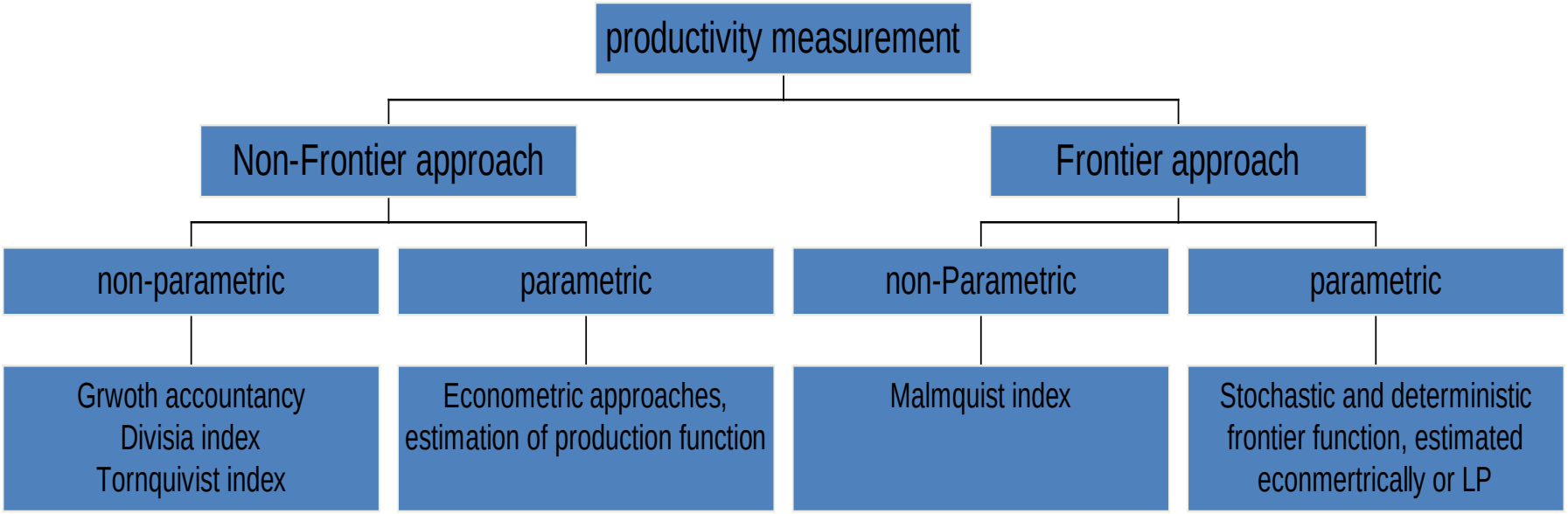
inflexible

States with inflexible labour market has not performed relatively worse in terms of TFP growth



Alternate approaches

Approaches to productivity measurement and analysis



Which technique to use?

Method

- **Translog TFP Index**
- Multilateral TFP Index
- Malmquist Index
- Levinsohn-Petrin methodology (or other methodology of same type)
- Data envelopment analysis
- Stochastic frontier production function

Type of data with the researcher (examples)

- Annual time series for one industry for 10 years
- Annual time series for 10 industries for15 years

Which technique to use?

Method

- Translog TFP Index
- **Multilateral TFP Index**
- **Malmquist Index**
- Levinsohn-Petrin methodology (or other methodology of same type)
- Data envelopment analysis
- Stochastic frontier production function

Type of data with the researcher

- Cross section data for one industry for 25 states, one year
- Panel data for one industry covering 25 states for 10 years

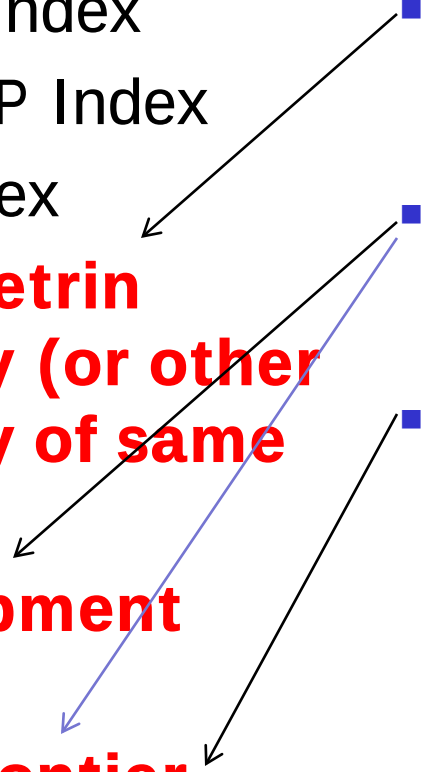
Which technique to use?

Method

- Translog TFP Index
- Multilateral TFP Index
- Malmquist Index
- **Levinsohn-Petrin methodology (or other methodology of same type)**
- **Data envelopment analysis**
- **Stochastic frontier production function**

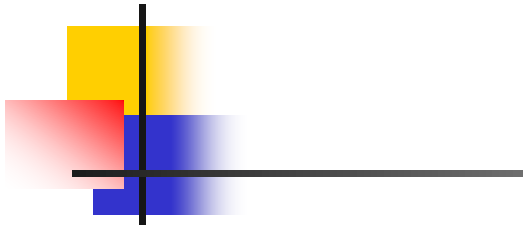
Type of data with the researcher

- Panel data for for 3000 factories belonging to one industry for 10 years
- Cross section data on 30 factories belonging to one industry for one year
- Cross section data for 3000 factories belonging to one industry for one year



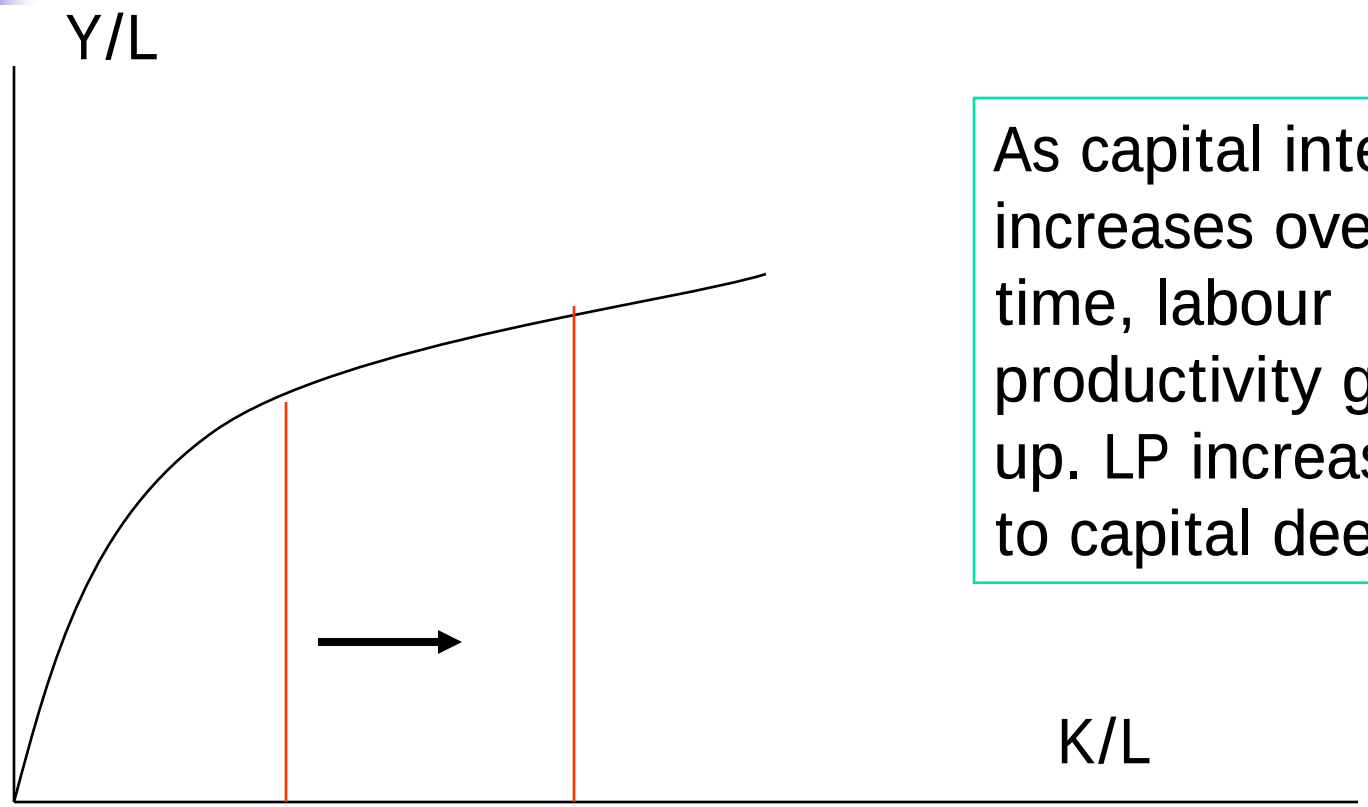
Productivity level versus productivity growth

- Some methods give the growth rate in productivity, some give the level of productivity.
- Some methods give both the level and growth rate in productivity.



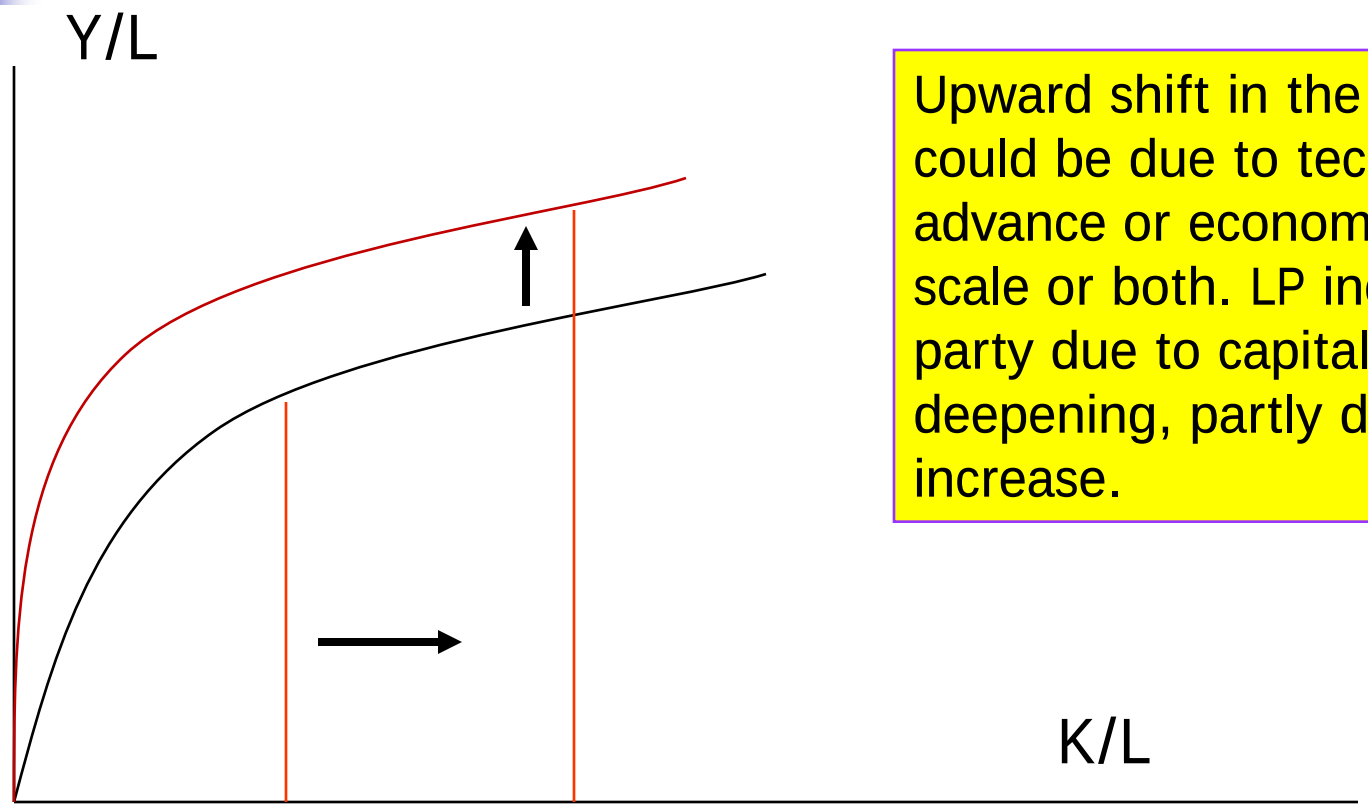
Conceptual framework

Relation between LP and KI



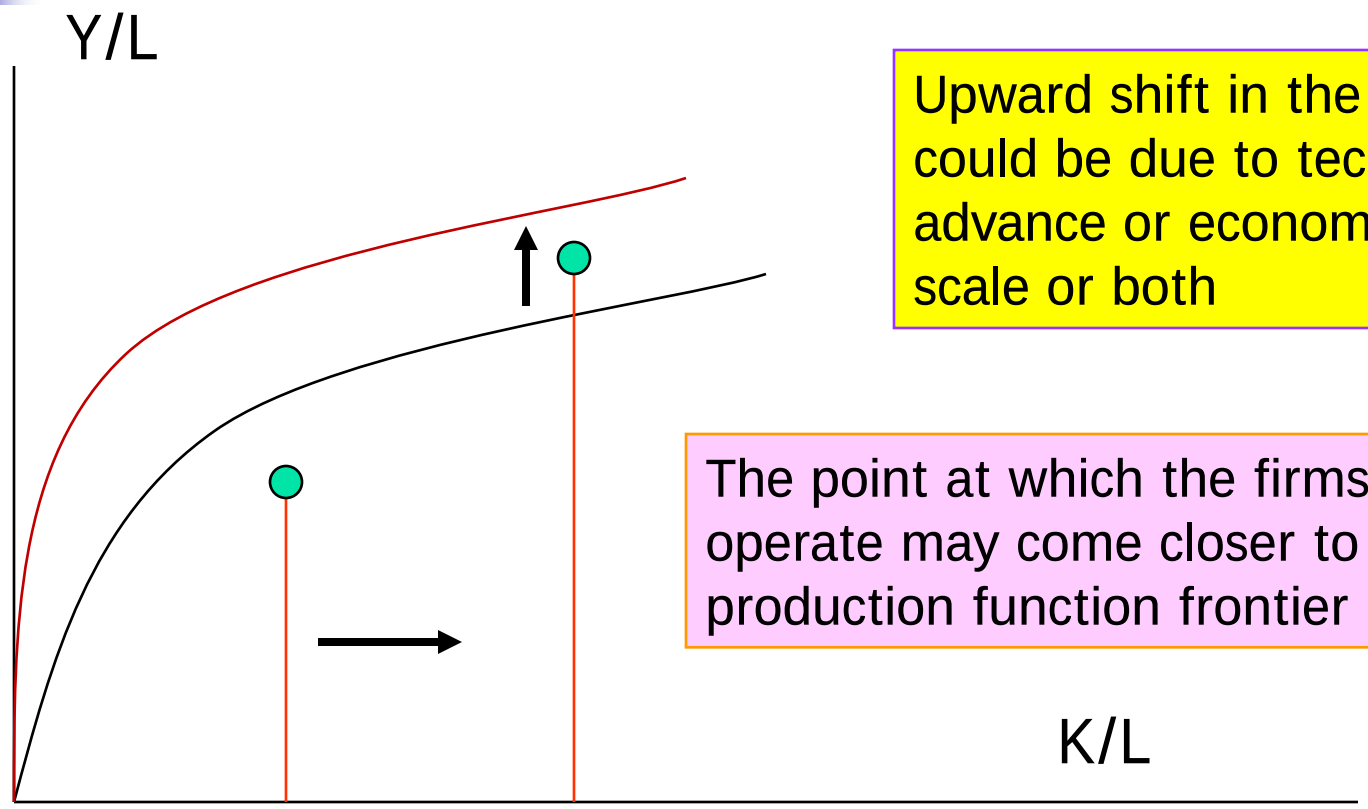
As capital intensity increases over time, labour productivity goes up. LP increase due to capital deepening

Relation between LP and KI



Upward shift in the curve could be due to technological advance or economies of scale or both. LP increase is party due to capital deepening, partly due to TFP increase.

Relation between LP and KI

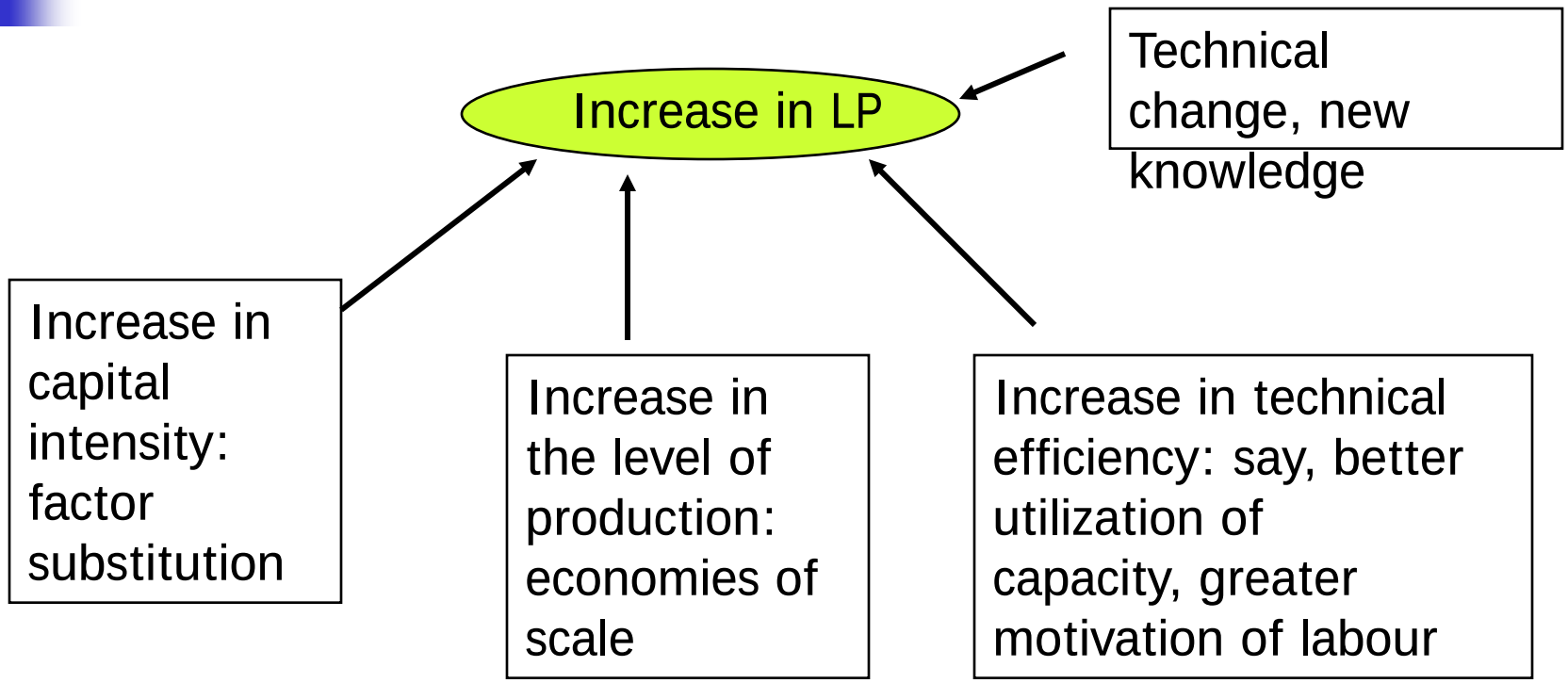


Upward shift in the curve could be due to technological advance or economies of scale or both

The point at which the firms operate may come closer to the production function frontier

Now there are three causes of increase in LP

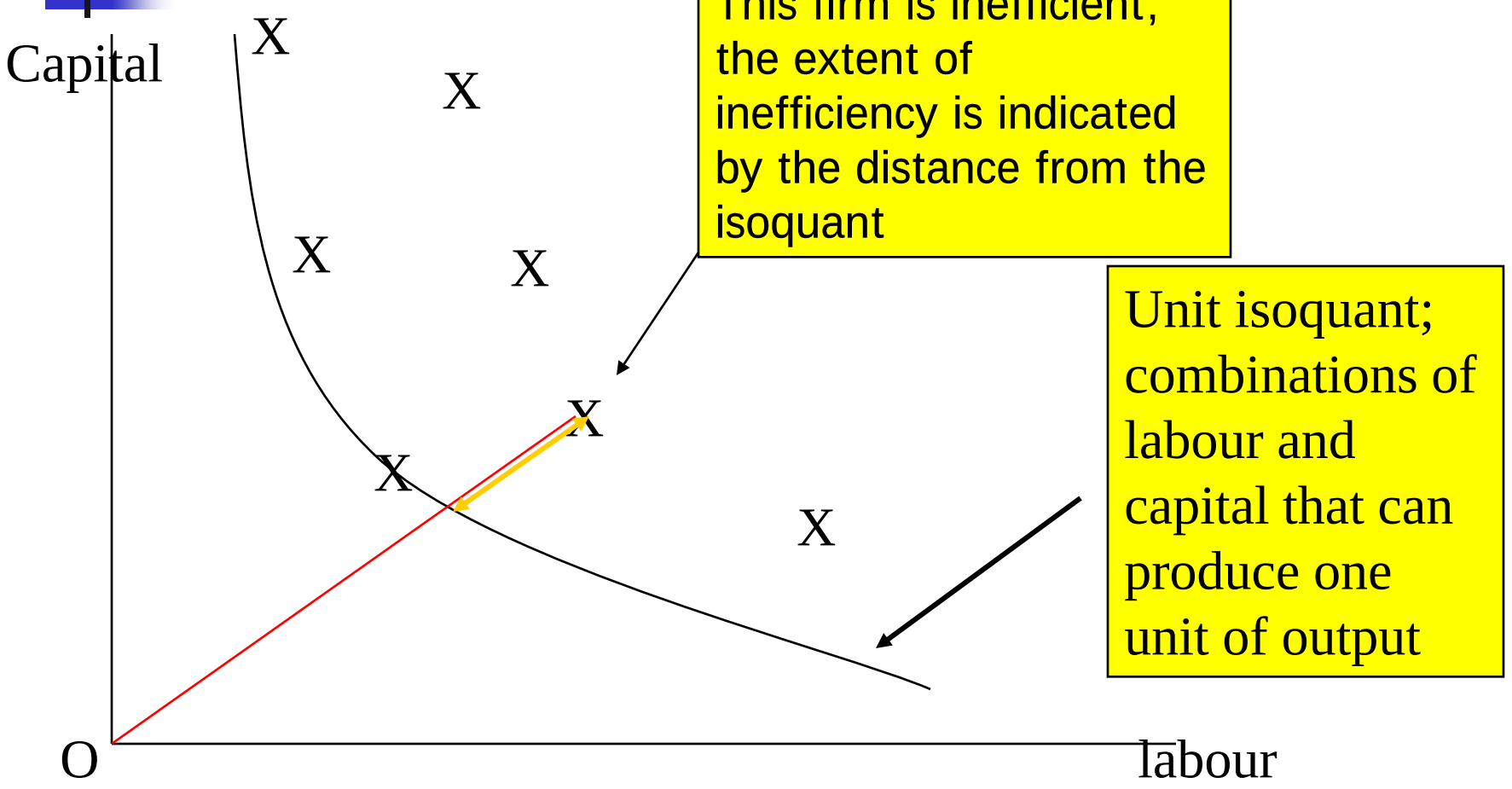
Sources of gain in labour productivity



Productivity and efficiency

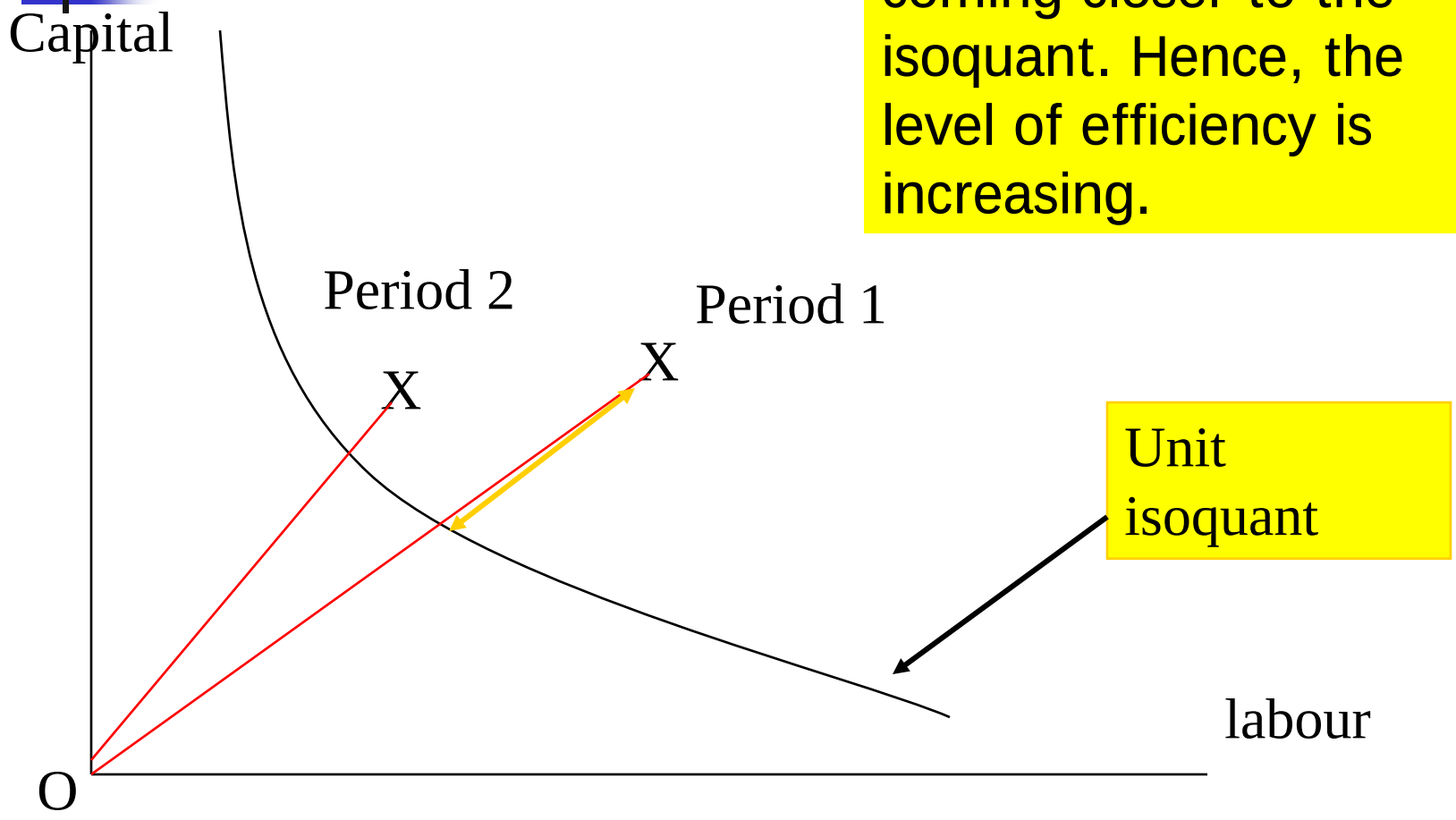
- Changes in productivity occur due to changes in technology and changes in efficiency (how well the technology is used). Thus, the rate of productivity growth is the sum of the rate of technical progress and the rate of improvement in technical efficiency.

Technical (in)efficiency

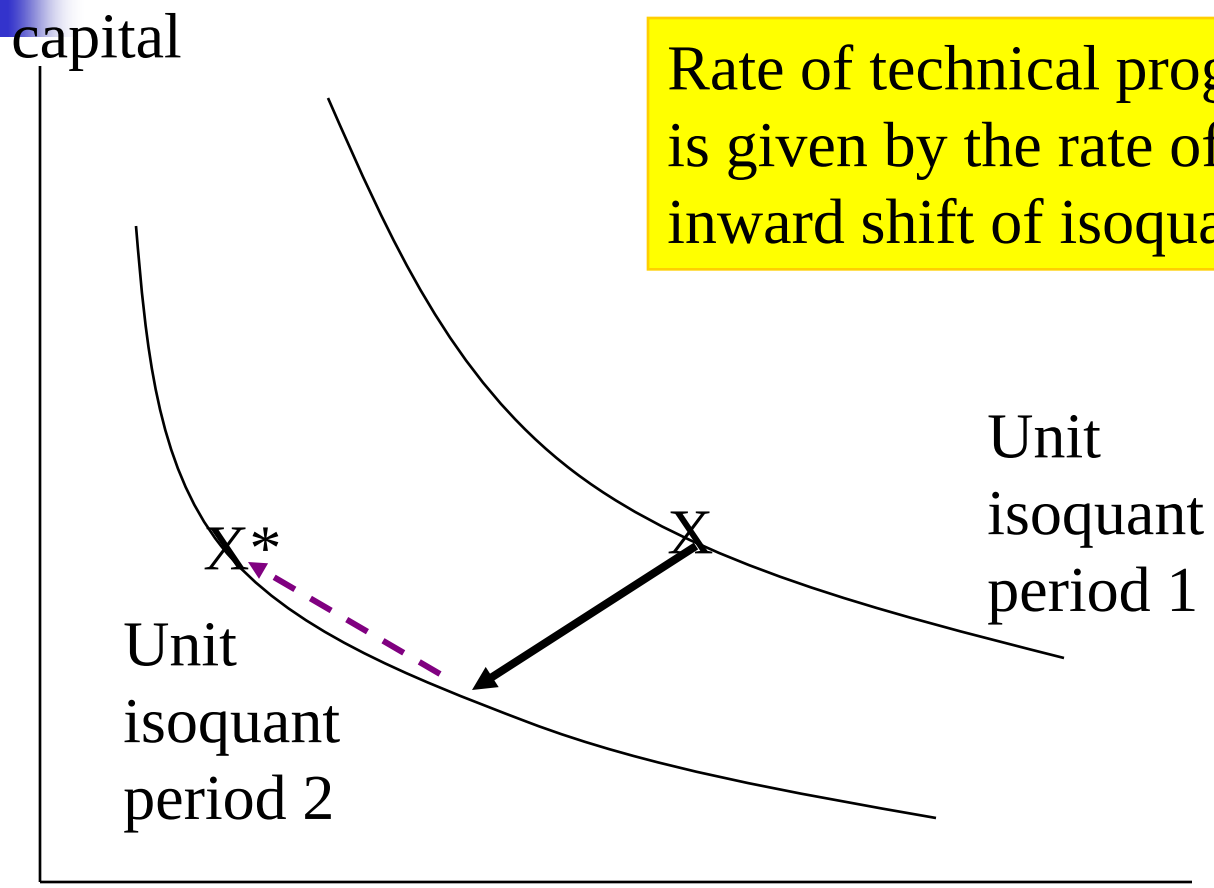


Improvement in technical efficiency

Over time, the firm is coming closer to the isoquant. Hence, the level of efficiency is increasing.



Technical change (progress)

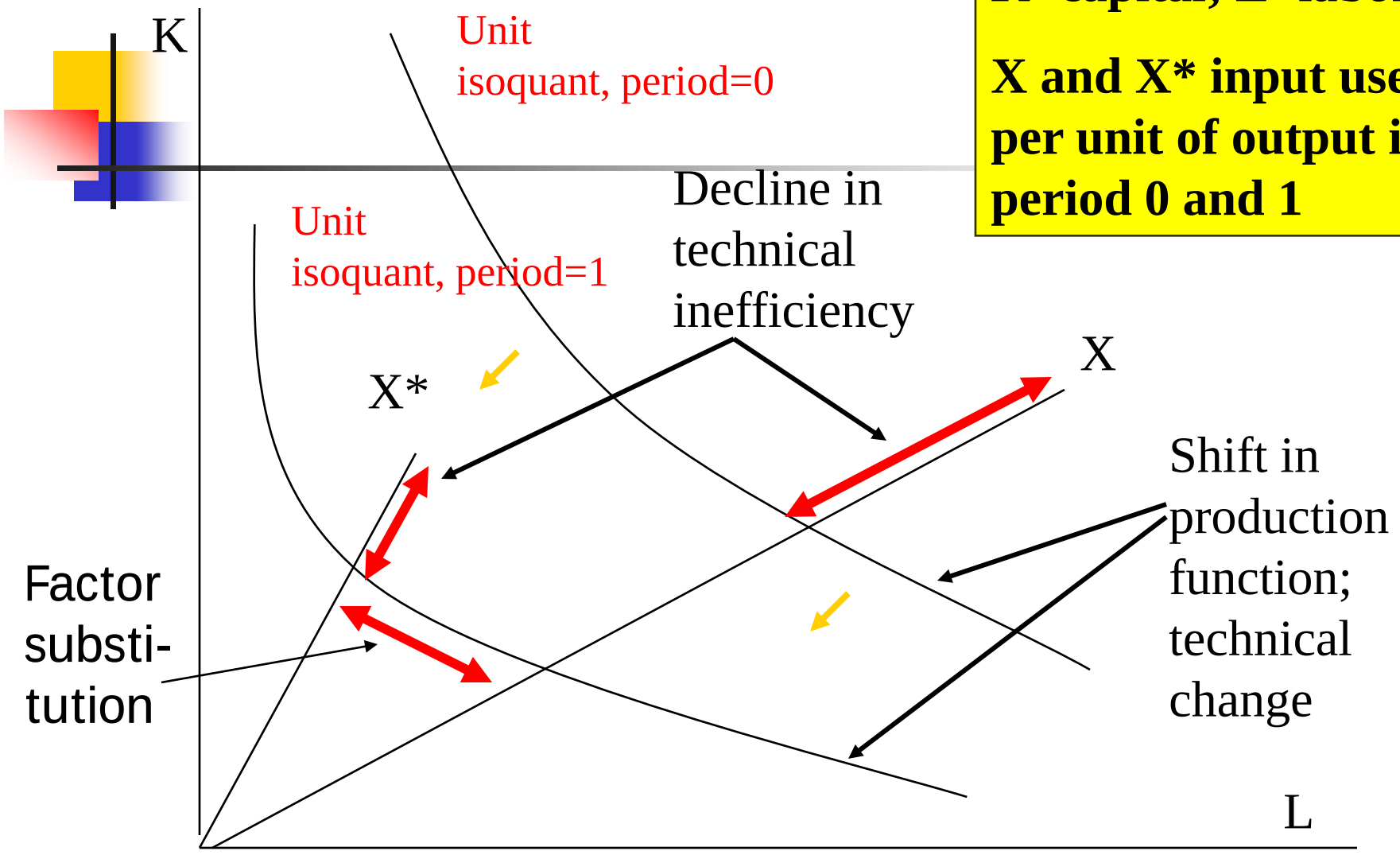


Rate of technical progress is given by the rate of inward shift of isoquant

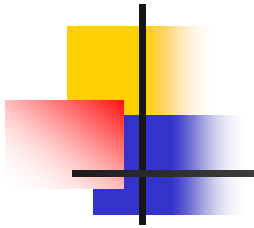
Tornqvist index assumes that the firms are always on the isoquant

K=capital; L=labor

X and X* input use per unit of output in period 0 and 1



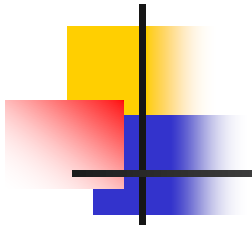
Q: How do we split, the movement from X to X* into the three parts?



Total factor Productivity – Divisia Index, Tornqvist index (Growth accounting)

Measurement of Total Factor Productivity - Growth accounting

- Growth accounting: In this approach, TFP indices are used for measuring TFP.
- TFP is defined as Y/I, where Y is the index of output, and I is the index of total input.
- The income shares of the factors of production are used as weights to compute the growth in total input.
- Similarly, when there are several output, revenue shares are used to combine growth of different outputs to form the growth rate in total output.



One output, three-input case (gross output function framework)

Growth accounting equation

- Y=f(L, K, X; t) : production function
- Taking total derivative

$$\frac{dY}{dt} = \frac{\partial Y}{\partial L}\frac{dL}{dt} + \frac{\partial Y}{\partial K}\frac{dK}{dt} + \frac{\partial Y}{\partial X}\frac{dX}{dt} + \frac{\partial Y}{\partial t}$$

- Dividing by Y

$$\frac{dY}{dt}\frac{1}{Y} = \frac{\partial Y}{\partial L}\frac{dL}{dt}\frac{1}{Y} + \frac{\partial Y}{\partial K}\frac{dK}{dt}\frac{1}{Y} + \frac{\partial Y}{\partial X}\frac{dX}{dt}\frac{1}{Y} + \frac{\partial Y}{\partial t}\frac{1}{Y}$$

Y = output (real); L = labor input; K = capital input; X = intermediate input; t=time

Growth accounting equation(2)

- One can derive

$$\frac{dY}{dt}\frac{1}{Y} = \frac{\partial Y}{\partial L}\frac{dL}{dt}\frac{1}{Y}\frac{L}{L} + \frac{\partial Y}{\partial K}\frac{dK}{dt}\frac{1}{Y}\frac{K}{K} + \frac{\partial Y}{\partial X}\frac{dX}{dt}\frac{1}{Y}\frac{X}{X} + \frac{\partial Y}{\partial t}\frac{1}{Y}$$

$$\frac{d\ln Y}{dt} = \frac{\partial Y}{\partial L}\frac{d\ln L}{dt}\frac{L}{Y} + \frac{\partial Y}{\partial K}\frac{d\ln K}{dt}\frac{K}{Y} + \frac{\partial Y}{\partial X}\frac{d\ln X}{dt}\frac{X}{Y} + \frac{\partial \ln Y}{\partial t}$$

$$\frac{d\ln Y}{dt} = S_L\frac{d\ln L}{dt} + S_K\frac{d\ln K}{dt} + S_X\frac{d\ln X}{dt} + \frac{\partial \ln Y}{\partial t}$$

TFP growth

Y = output (real); L = labor input; K = capital input; X= intermediate inputs; SL = income share of labor; SX= income share of int. input, and SK = inc. share of capital (= 1-SL-SX)

Tornqvist index/ Translog index of TFP

The Translog index of TFP is commonly used for measuring TFP growth. It does not require the assumption of neutral technical change and allows for variable elasticity of substitution. It may be written as:

$$\Delta \ln TFP(t) = \Delta \ln Y(t) - [\frac{SL(t) + SL(t-1)}{2} \times \Delta \ln L(t)] - [\frac{SK(t) + SK(t-1)}{2} \times \Delta \ln K(t)]$$
$$- [\frac{SX(t) + SX(t-1)}{2} \times \Delta \ln X(t)]$$

TFP = total factor productivity; Y = output (real)
L = labor input; K = capital input; X= intermediate input; SL = income share of labor, SX= income share of int. input; and SK = share of capital (= 1-SL-SX)

Compute in Excel

Translog index of TFP

Average income share of two years is taken

$$\Delta \ln TFP(t) = \Delta \ln Y(t) - [\frac{SL(t) + SL(t-1)}{2} \times \Delta \ln L(t)] - [\frac{SK(t) + SK(t-1)}{2} \times \Delta \ln K(t)]$$
$$- [\frac{SX(t) + SX(t-1)}{2} \times \Delta \ln X(t)]$$

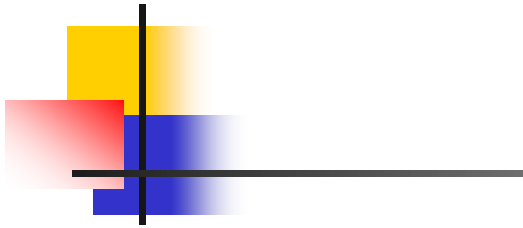
TFP = total factor productivity; Y = value added (real)

L = labor input; K = capital input; SL = income share of labor, SX= inc. share of int. input, SK = share of capital (= 1-SL-SX)

Growth rates in output, intermediate input, labor and capital

Translog/Tornqvist index - assumptions

- Constant returns to scale
- Competitive equilibrium – producers maximize profits – minimize cost – input combination is so chosen that marginal product of each factor is equal to its price
- Disembodied technical change – firms can take advantage of new technology without changing input use



One output, two-input case (value added function framework)

Growth accounting equation

- Y=f(L, K; t) : production function
- Taking total derivative

$$\frac{dY}{dt} = \frac{\partial Y}{\partial L}\frac{dL}{dt} + \frac{\partial Y}{\partial K}\frac{dK}{dt} + \frac{\partial Y}{\partial t}$$

Y = value added (real); L = labor input; K = capital input; t=time

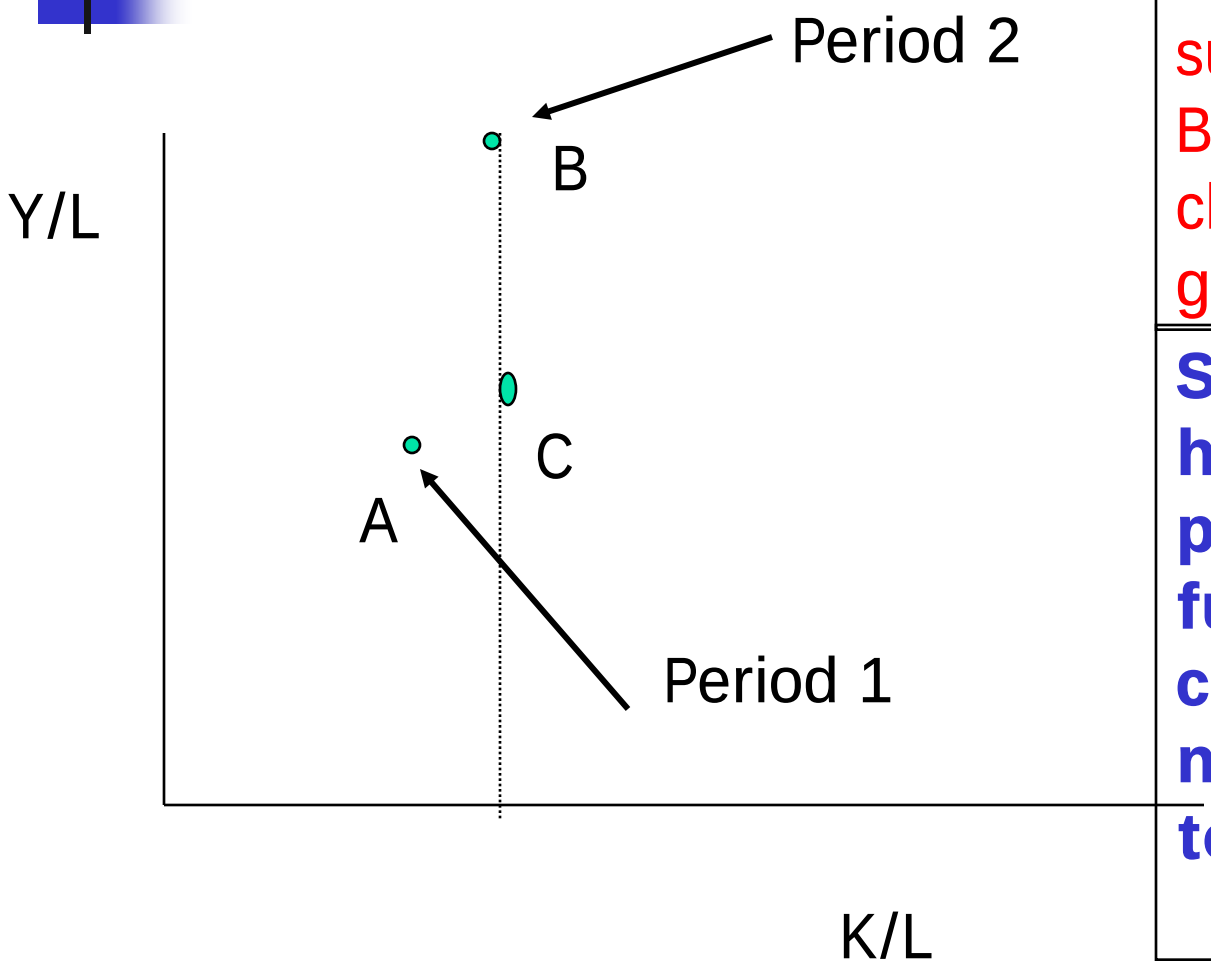
Tornqvist index/ Translog index of TFP

Compute in Excel

$$\Delta \ln TFP(t) = \Delta \ln Y(t) - [\frac{SL(t) + SL(t-1)}{2} \times \Delta \ln L(t)] - [\frac{SK(t) + SK(t-1)}{2} \times \Delta \ln K(t)]$$

TFP = total factor productivity; Y = value added (real)
L = labor input; K = capital input; SL = income share of labor; and SK = share of capital (= 1-SL)

Translog index of TFP - What is the problem, and how is that solved?

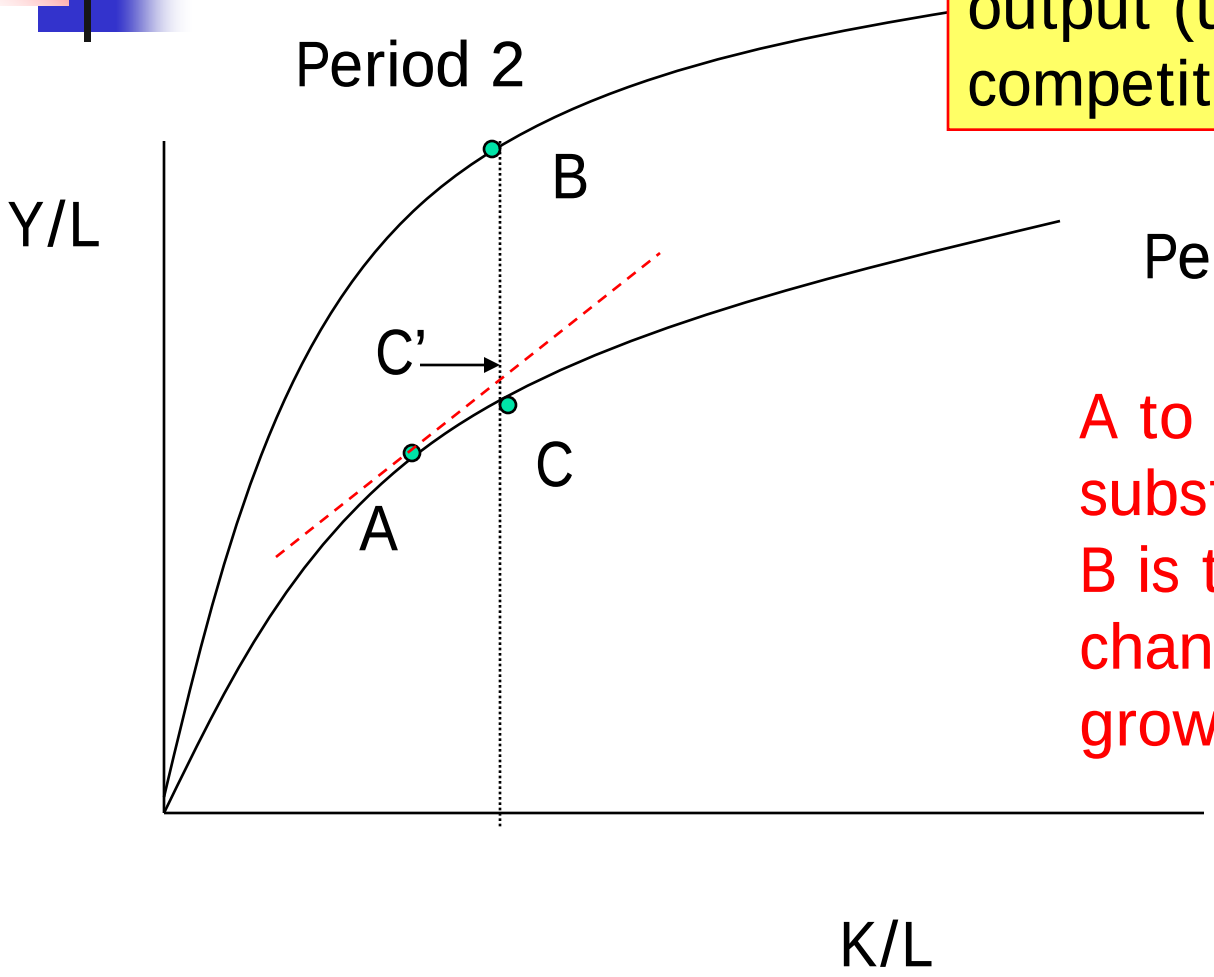


A to C is factor substitution; C to B is technical change, or TFP growth

Since we do not have the production function (i.e. the curves), we do not know where to fix point C

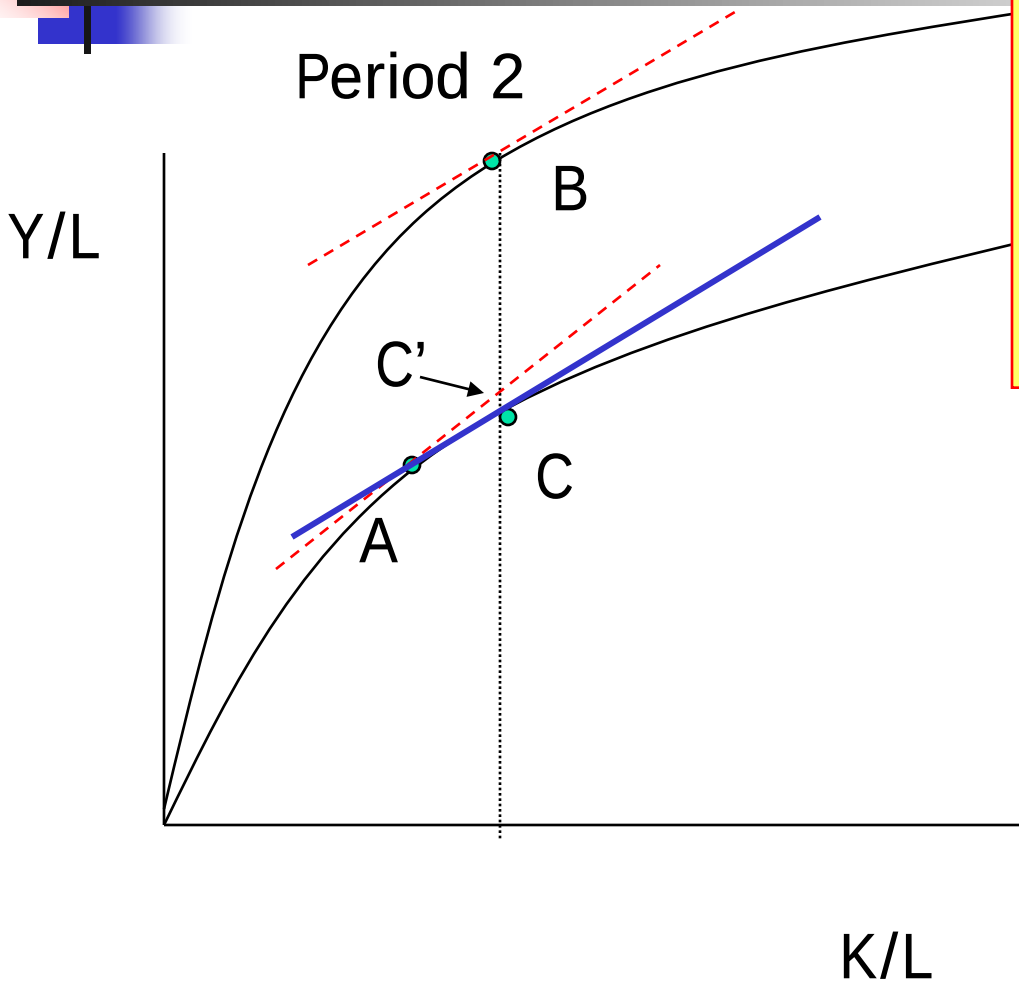
One way out

Use the slope at point A to approximate. Slope= income share of capital in output (under perfect competition)



A to C is factor substitution; C to B is technical change, or TFP growth

Better approximation



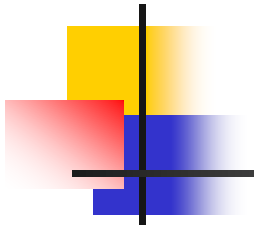
Take average of slopes at points A and B to approximate. Slope= income share of capital in output. This brings C' closer to true C. This is what Tornqvist index does.

A to C is factor substitution; C to B is technical change, or TFP growth

Application

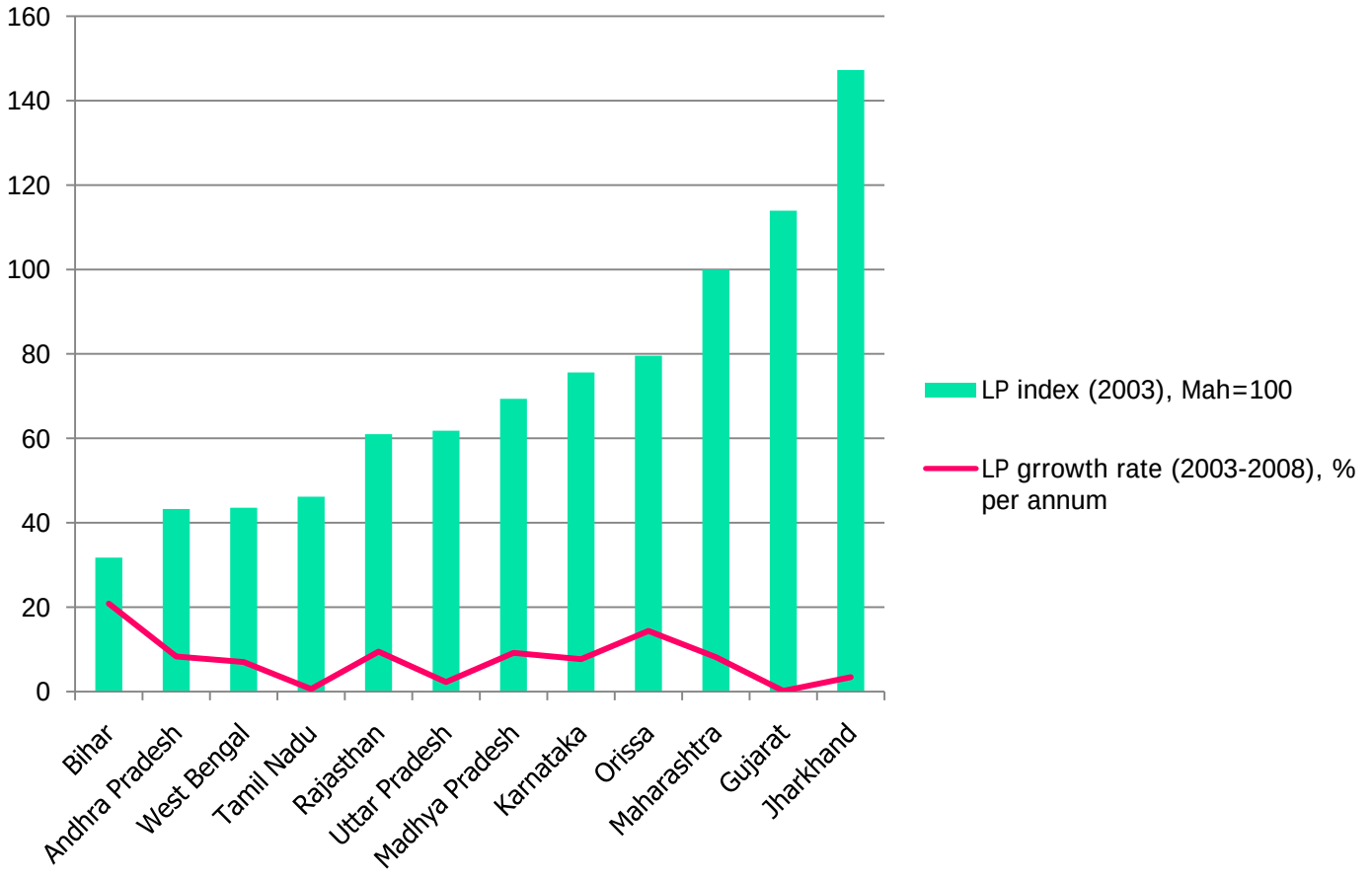
NIC	Description	TFP growth rate 1999 to 2011 (% p.a.)
15	Food products and beverages	1.15
16	Tobacco products	-2.87
30+32+33	Electronics, computers, mobiles	12.19
34	Motor vehicles	2.53
	All manufacturing	1.70

B. Goldar, Productivity in Indian Manufacturing (1999-2011): accounting for imported materials input, EPW, August 29, 2015. Five inputs considered: capital, labour, energy, materials, services (KLEMS).



Multilateral total factor productivity index

Labour productivity, level and growth, Org mfg by state



Multilateral total factor productivity index

- If one wants to study variations in TFP across regions (or firms) and over time, then a multilateral TFP index can be used.
- If there are only two inputs, labour and capital, the multilateral TFP index may be written as:

Compute in Excel

Multilateral TFP Index

$$TFP_{bc} = \left(\frac{Y_b}{Y_c}\right)\left(\frac{\bar{L}}{L_b}\right)^{\alpha_b}\left(\frac{\bar{K}}{K_b}\right)^{\beta_b}\left(\frac{L_c}{\bar{L}}\right)^{\alpha_C}\left(\frac{K_c}{\bar{K}}\right)^{\beta_c}$$

The index expresses the productivity level in region-year b (say, UP in 2004) as a ratio to the productivity level in region-year c (say, Punjab in 2000). L and K with bar are sample average (geometric mean). The coefficients represent incomes shares of labour and capital.

Weights in the index

- Let SL_b be the income share of labour in state-year b and $\overline{SL}$ the arithmetic mean of labour share in value added across all the observations. Then, α_b may be written as:

$$\alpha_b = \frac{SL_b + \overline{SL}}{2}$$

- α_c, β_b and β_c may be defined in a similar way.

What Multilateral TFP Index compares?

	State 1	State 2	State 3	State 4
Year 1	c		b	
Year 2				b
Year 3		b		
Year 4	b			

Assumptions Underlying the Index

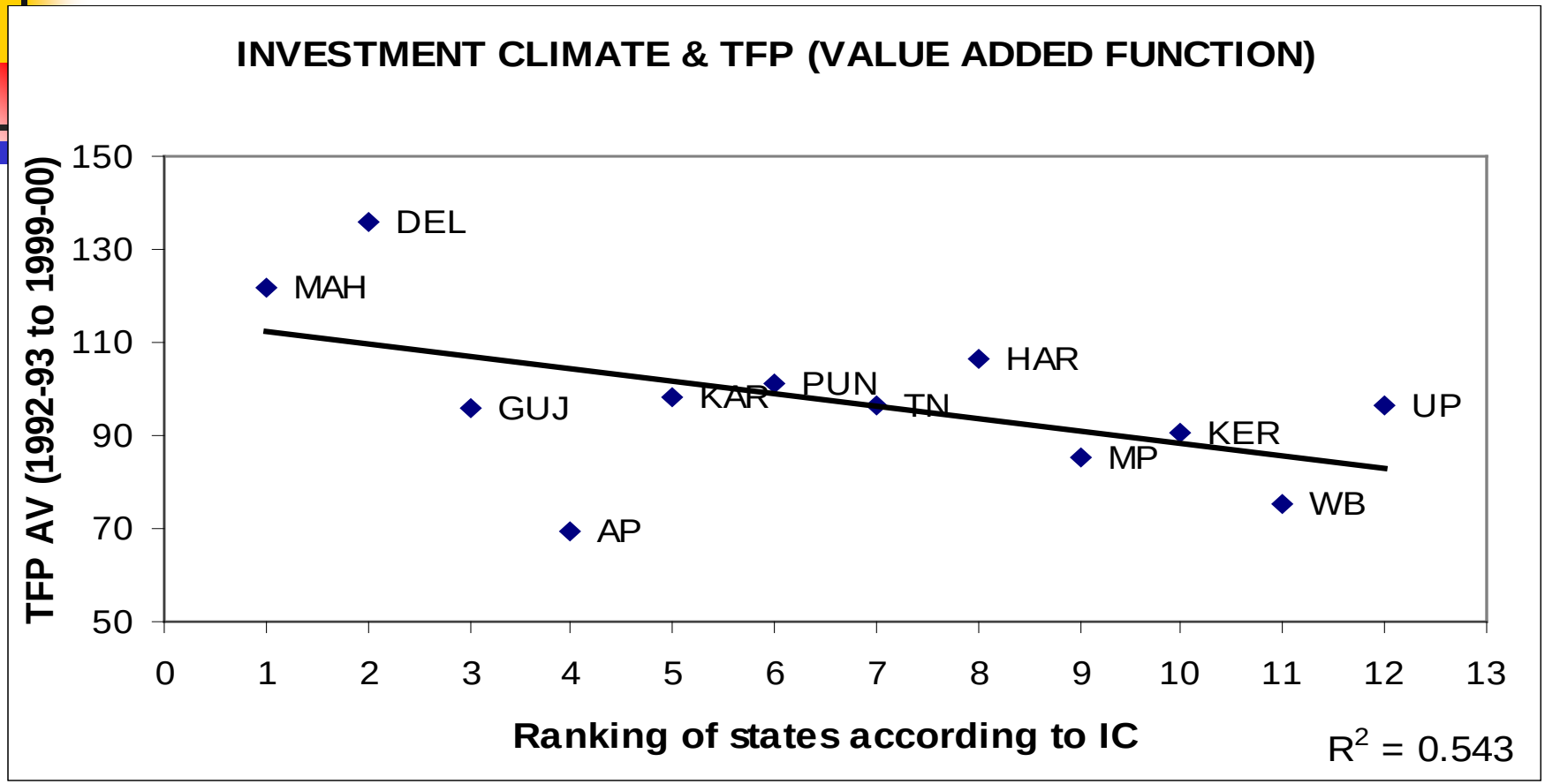
- Constant returns to scale
- Factors paid according to marginal product

References

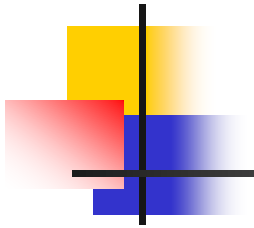
- Basic methodology: **D. Caves, L. R. Christensen, and D. E. Diewert**, "Multilateral comparisons of output, input and productivity using superlative index numbers." *Economic Journal*, 92: 73-86, 1982.
- Application to Indian industry, state-wise comparison: **C. Veeramani and B. Goldar,** "Manufacturing Productivity in Indian States: Does Investment Climate Matter?" *Economic and Political Weekly*, June 11, 2005.

Multi-lateral TFP index (MAH, 1998=1.00)

	1998-99	2003-04
Andhra Pradesh	0.60	0.77
Gujarat	0.71	0.96
Haryana	0.77	1.14
Karnataka	0.64	0.94
Maharashtra	1.00	1.17
Orissa	0.57	0.72
Tamil Nadu	0.70	0.79
Uttar Pradesh	0.57	0.89
West Bengal	0.76	0.72



Regression line is fitted and R^2 is computed without including Andhra Pradesh. **C. Veeramani and B. Goldar, 2005, EPW**

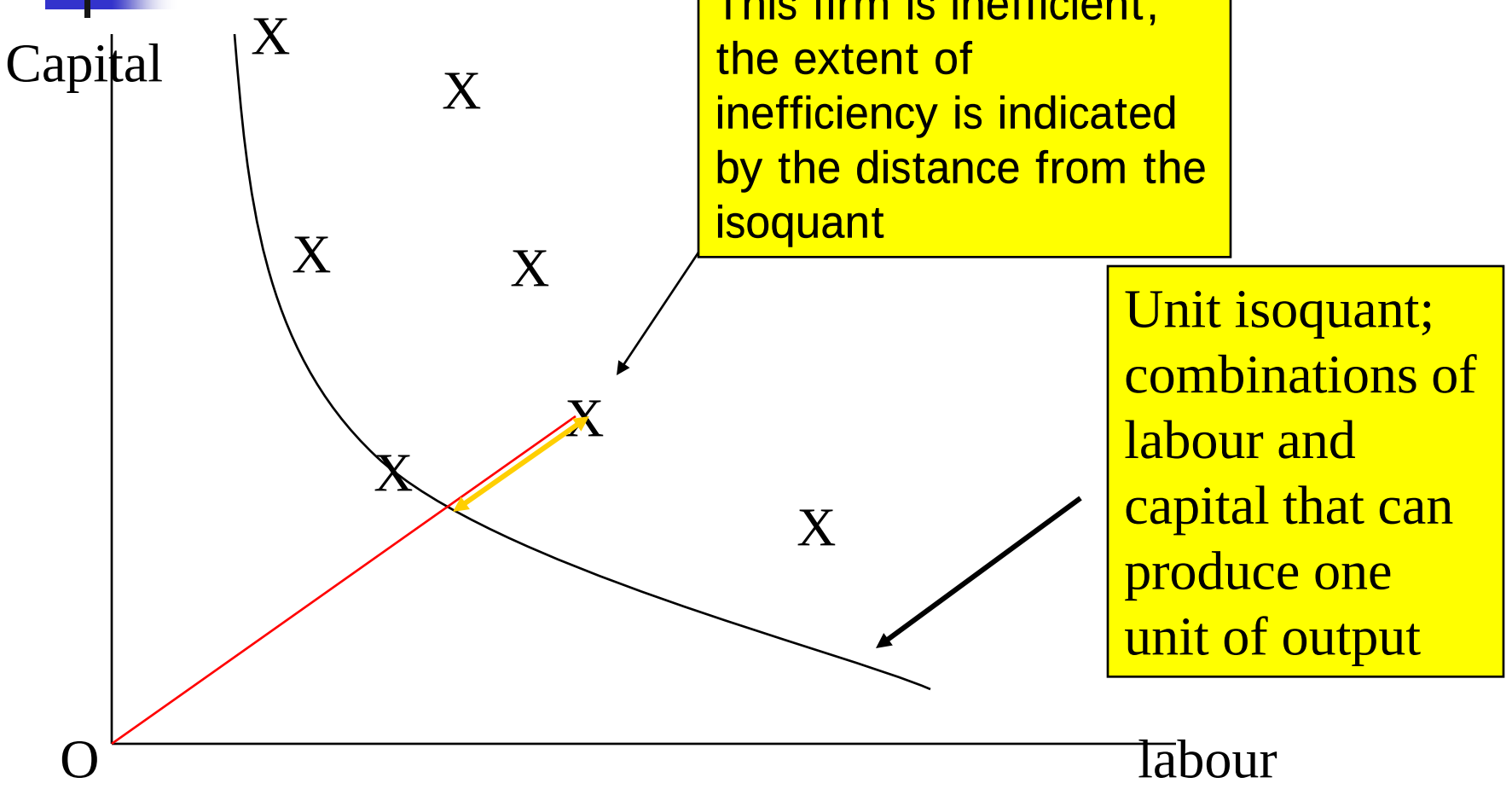


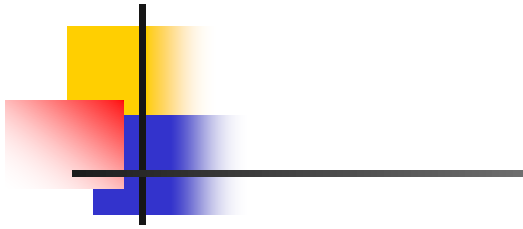
Technical Efficiency, Concept and measurement

Efficiency

- Efficiency (or technical efficiency to be more specific) may be defined as the **ratio of actual output to the potential/optimal output from a given bundle of inputs and given technology**.

Technical (in)efficiency





Data Envelopment Analysis (DEA)

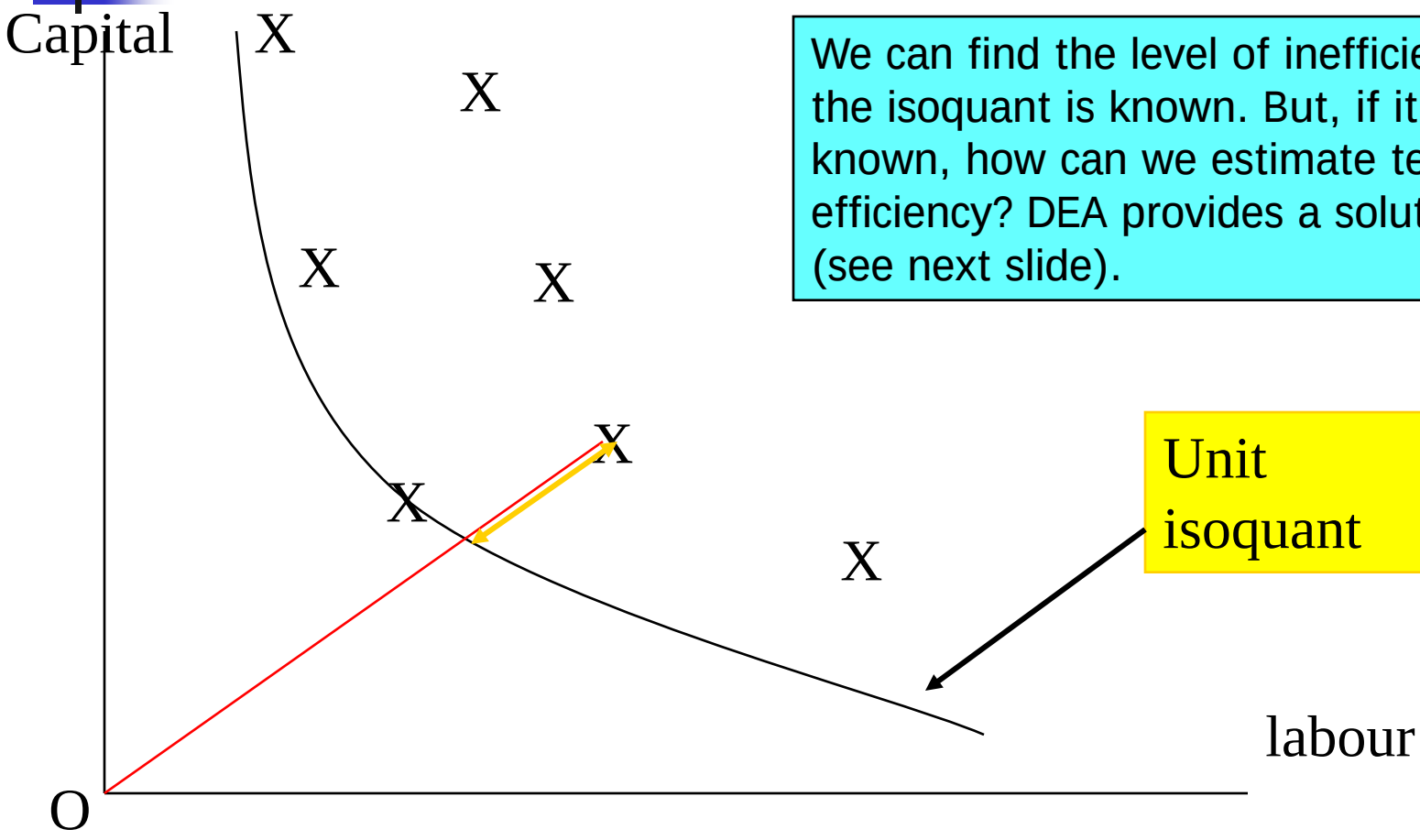
DEA

- Data envelopment analysis (DEA) is widely used for studying technical efficiency in cross-sectional context.

DEA

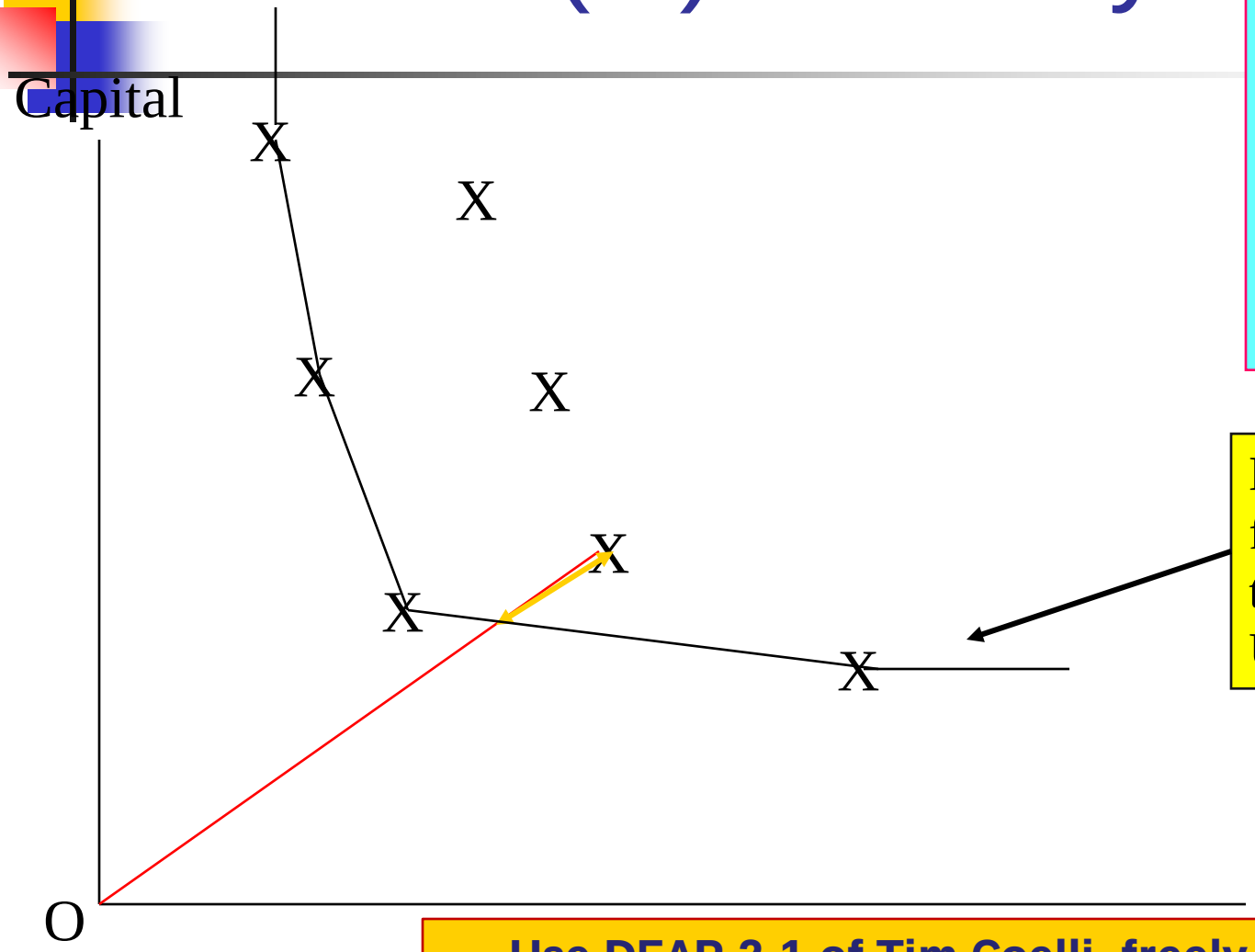
- Main advantage of DEA: it does not impose any functional form on the production function. Based on the observed input-output points, the best practice production frontier is derived and firms/regions are compared to the best practice frontier.
- Another advantage is that this methodology does not need data on input (or output) prices.
- The disadvantage is that the results are sensitive to outliers.

Technical (in)efficiency



We can find the level of inefficiency, if the isoquant is known. But, if it is not known, how can we estimate technical efficiency? DEA provides a solution (see next slide).

Technical (in)efficiency

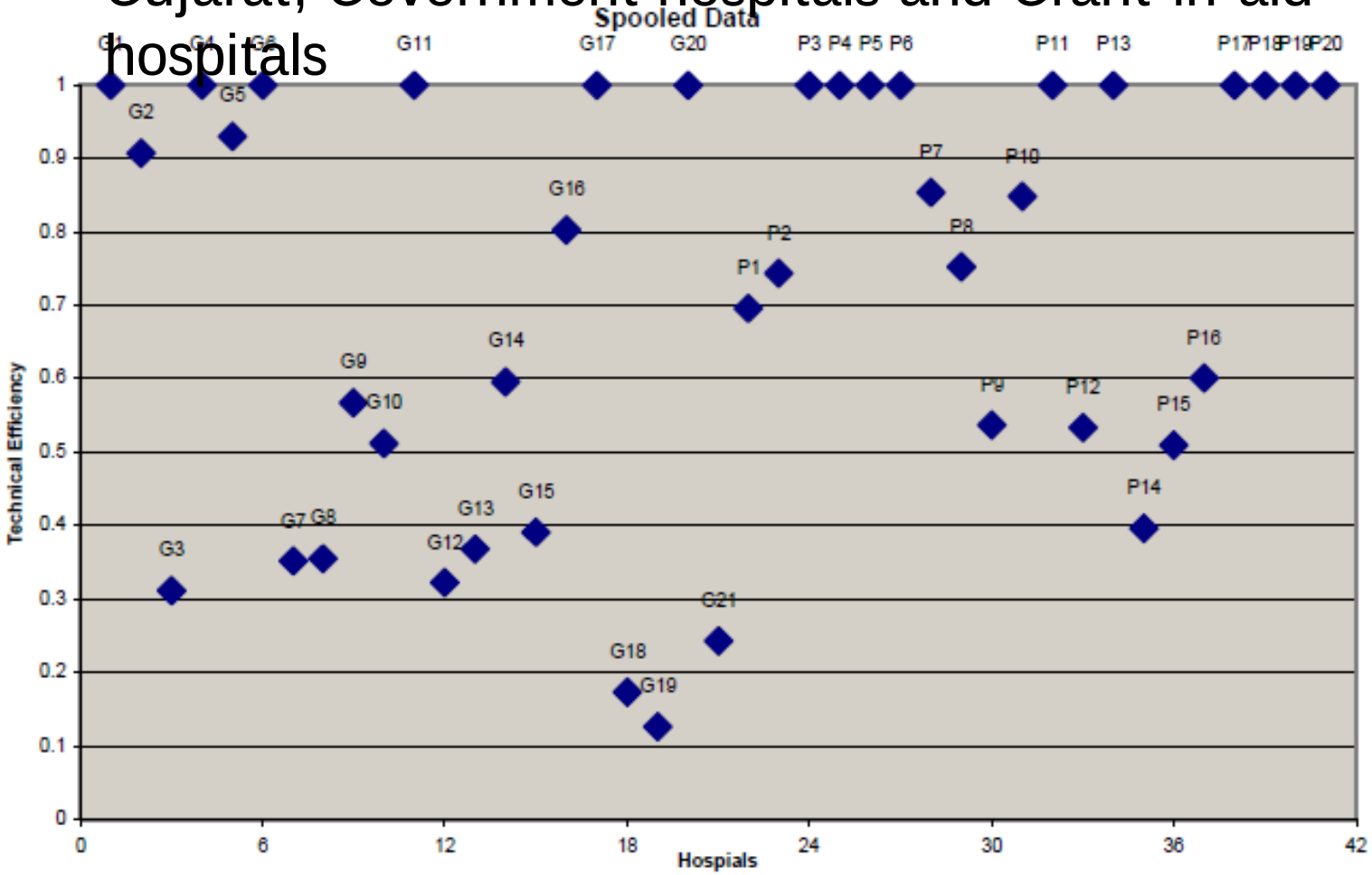


Software, pric ed and free, are available to estimate DEA frontier and technical efficiency

DEA frontier, taken to represent Unit isoquant

Use DEAP 2.1 of Tim Coelli, freely downloadable for calculating TE

Study of hospital efficiency in Gujarat, Government hospitals and Grant-in-aid hospitals



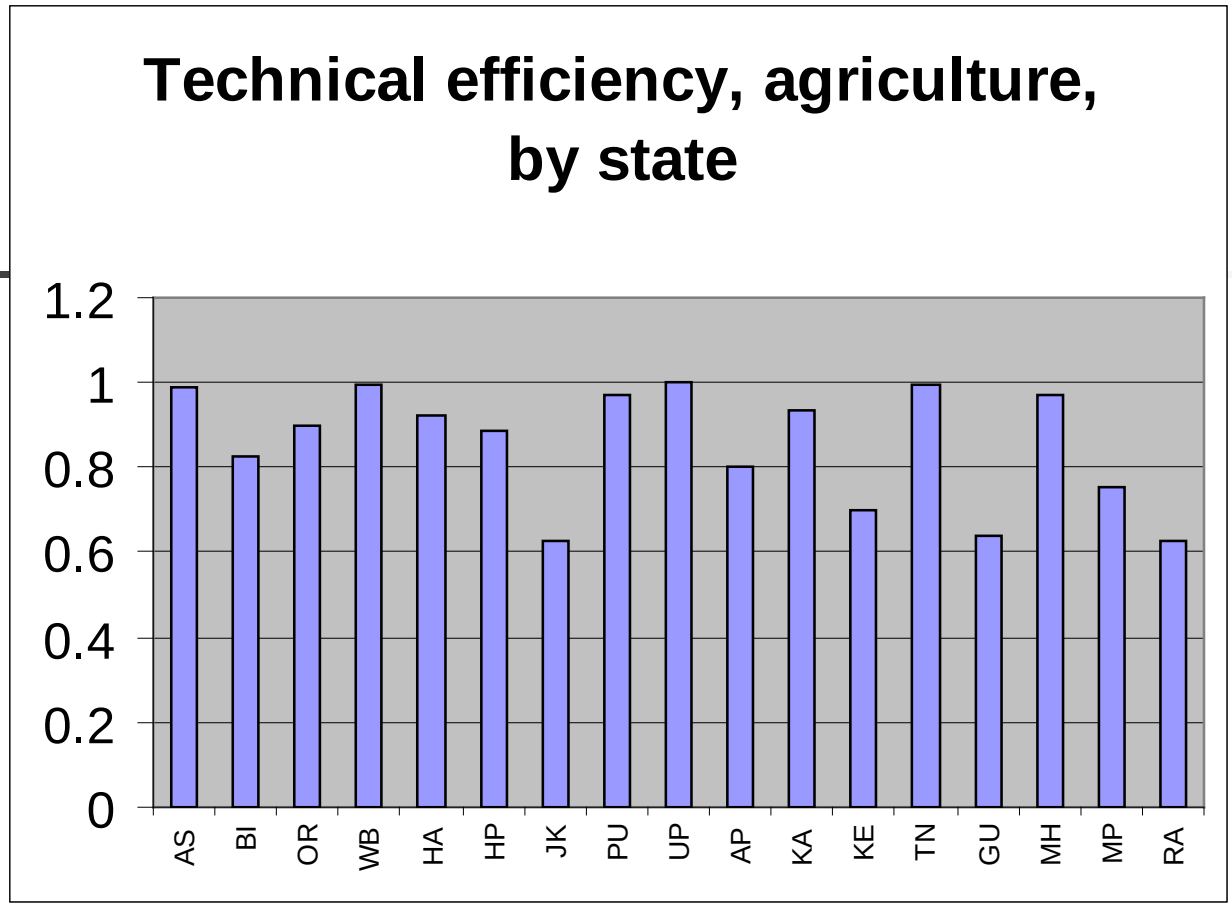
An empirical analysis of district hospitals and grant-in-aid hospitals in Gujarat state of India, **Ramesh Bhat**, **Bharat Bhushan Verma**, and **Elan Reuben**, July 2001, Indian Institute of Management, Ahmedabad

Methodological issue: Radial and non-radial efficiency

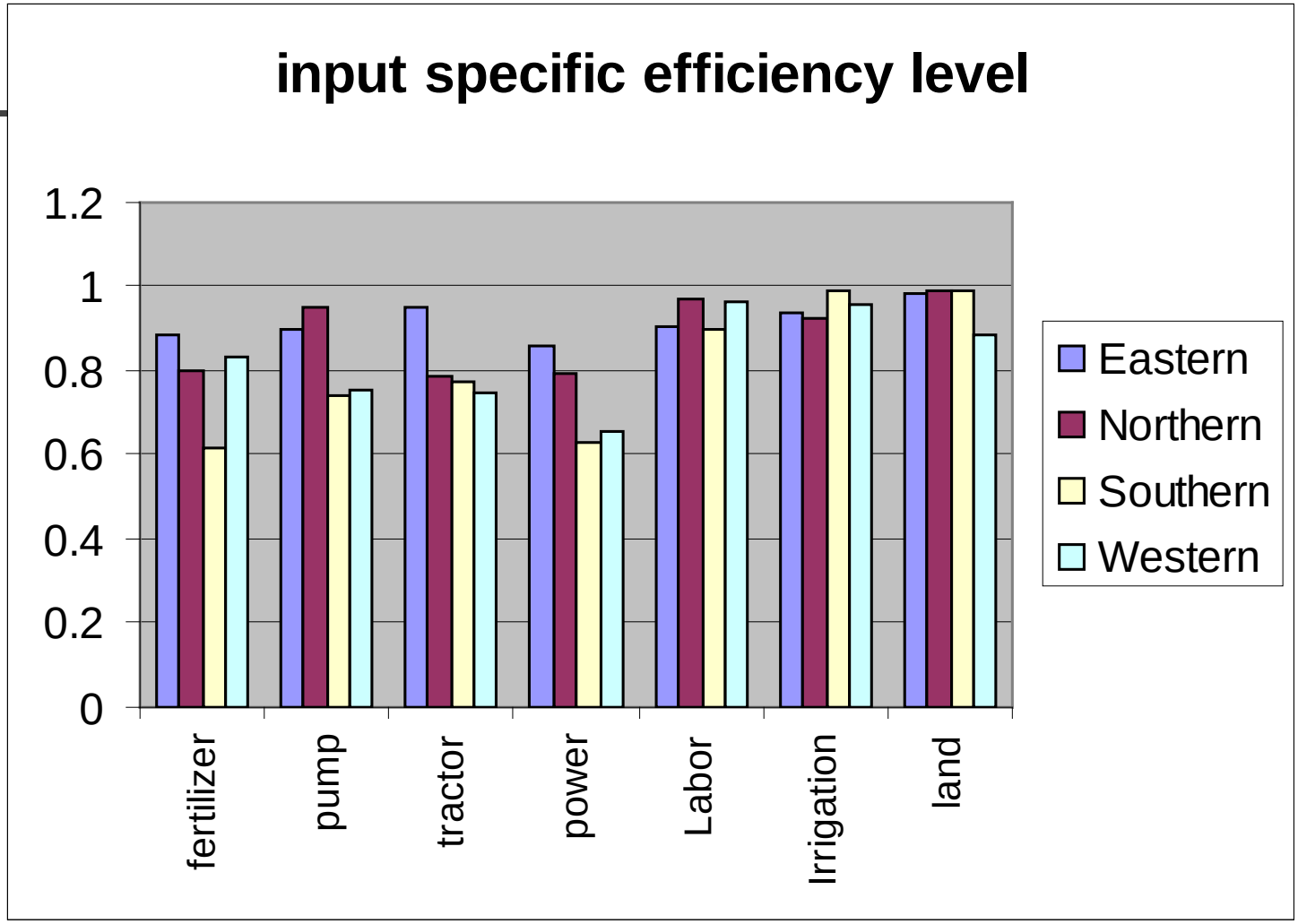
- The methods described above provide radial efficiency, which assume that all inputs are reduced proportionately. But, one may use other measures that do not require proportionate reduction.
- One possibility is to use measures that take care of slack – Slack adjusted radial measure of technical efficiency

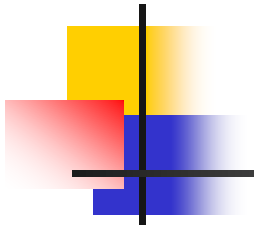
Non-radial measure

- Here we consider the maximum reduction possible in each input. The reductions are so chosen that the average improvement across inputs is the best. This measure is due to Fare and Lovell; Fare, Lovell and Zieschang.
- The advantage of this methodology is that the degree of inefficiency in respect of each input can be computed.

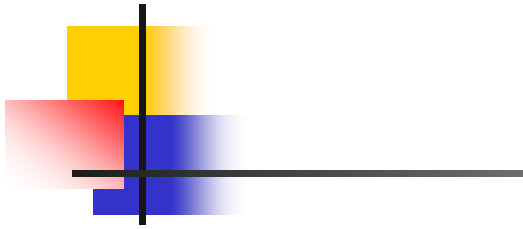


Production Efficiency in Indian Agriculture: An Assessment of the Post Green Evolution Years, Subhash C. Ray, University of Connecticut and Arpita Ghose, Jadavpur University Working Paper 2010-26, *Department of Economics Working Paper Series,* University of Connecticut, October 2010.


input specific efficiency level
1.2
1
0.8
0.6
0.4
0.2
0
fertilizer
pump
tractor
power
Labor
Irrigation
land
Eastern
Northern
Southern
Western



Total Factor Productivity, Technical Efficiency and Technical Change – Malmquist index



Distance function

Representation of technology

- For each time period t=1, …, T, the production technology $\mathbf{S^t}$ models the transformation of inputs, $\mathbf{x^t} \in \Re_+^n$ into outputs, $\mathbf{y^t} \in \Re_+^m$

$$\mathbf{S^t = \{(x^t, y^t): x^t \text{ can produce } y^t \}}$$

S is a set of observations of x and y such that x bundle of inputs can produce output y

Definition of distance function

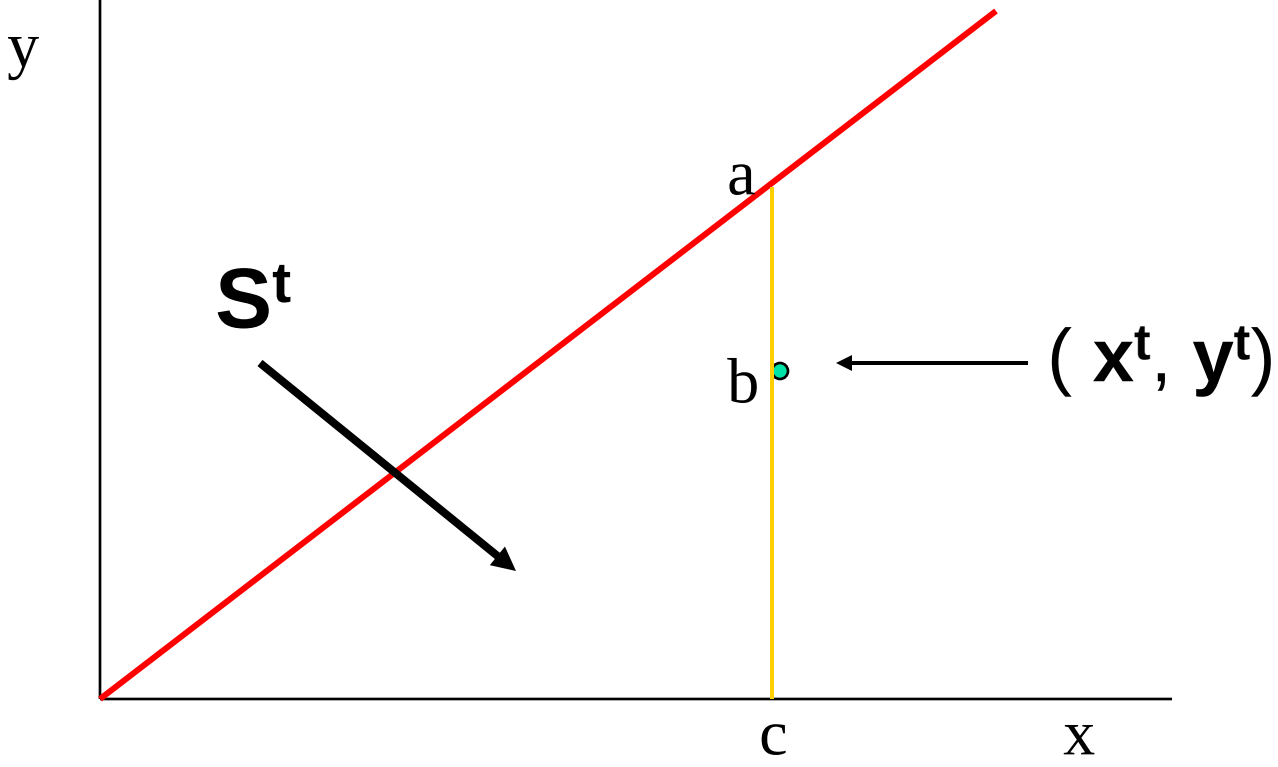
- The output distance function is defined at t as:

$$D_o^t(\mathbf{x}^t, \mathbf{y}^t) = \inf\{\theta: (\mathbf{x}^t, \mathbf{y}^t/\theta) \in \mathbf{S}^t\}$$

- $D_o^t(\mathbf{x}^t, \mathbf{y}^t) <= 1$ if and only if $(\mathbf{x}^t, \mathbf{y}^t) \in \mathbf{S}^t$
- $D_o^t(\mathbf{x}^t, \mathbf{y}^t) = 1$ if and only if $(\mathbf{x}^t, \mathbf{y}^t)$ is on the boundary or frontier of technology. This occurs when production is technically efficient

Case of single output and input

$\theta = bc/ac$



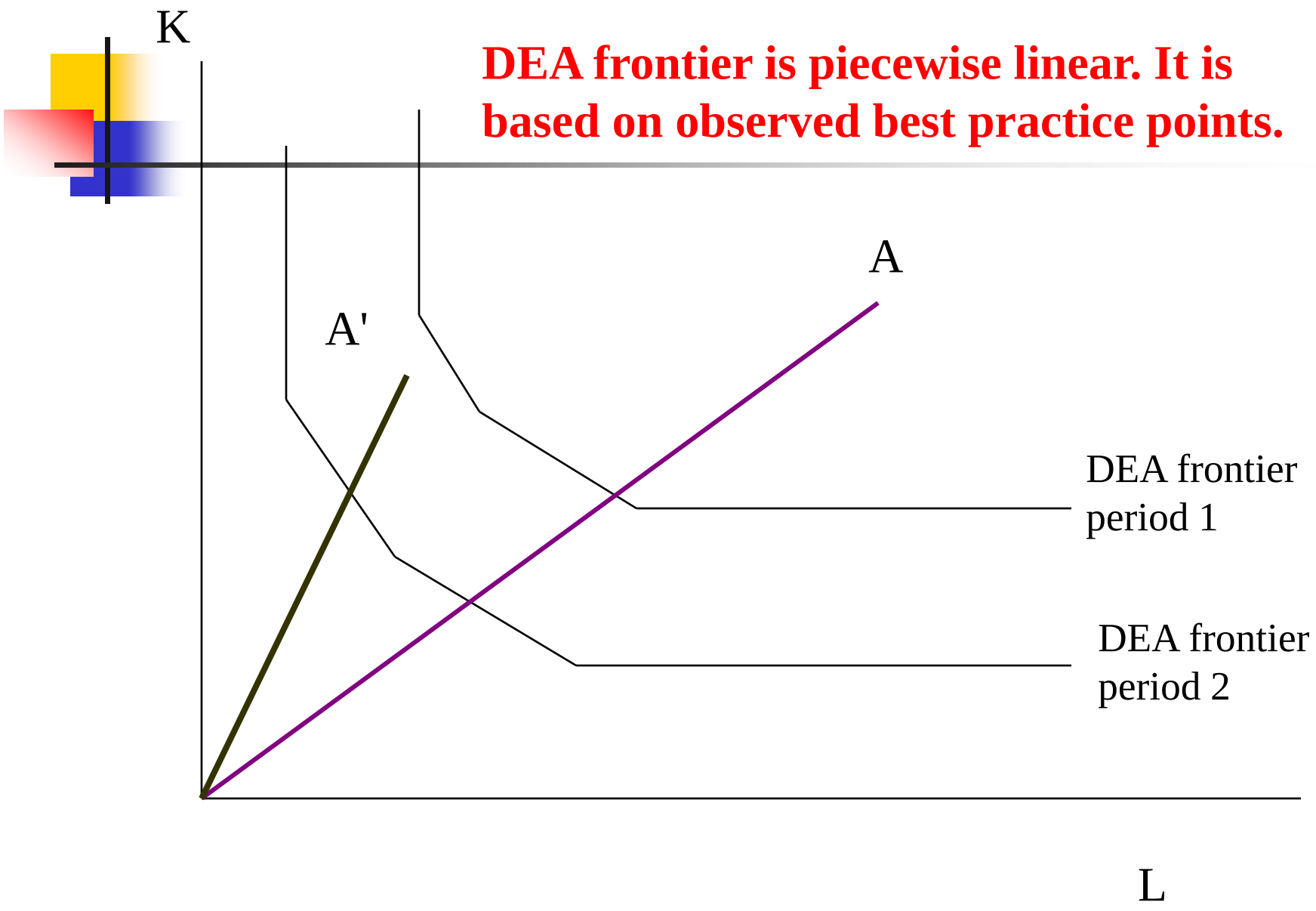
Production function: $y=\beta x$

Malmquist index

$$M^{t}(\mathbf{X}^{t}, \mathbf{Y}^{t}, \mathbf{X}^{t+1}, \mathbf{Y}^{t+1}) = \left[\frac{D^{t}(\mathbf{X}^{t+1}, \mathbf{Y}^{t+1})}{D^{t}(\mathbf{X}^{t}, \mathbf{Y}^{t})} \times \frac{D^{t+1}(\mathbf{X}^{t+1}, \mathbf{Y}^{t+1})}{D^{t+1}(\mathbf{X}^{t}, \mathbf{Y}^{t})} \right]^{\frac{1}{2}}$$

The Malmquist productivity-change index is formed by taking four distance functions.

DEA frontier is piecewise linear. It is based on observed best practice points.



- This involves four distance functions. Note here that one has to make comparison of observation for period 1 with frontier of period 1 and with the frontier of period 2. Similarly, the observation for period 2 is compared with the frontiers of periods 1 and 2.
- The expression can be broken down into two components: the change in technical efficiency and the technical change.

Malmquist index - decomposed

$$M^t(\mathbf{X}^t, \mathbf{Y}^t, \mathbf{X}^{t+1}, \mathbf{Y}^{t+1}) = \left[\frac{D^t(\mathbf{X}^{t+1}, \mathbf{Y}^{t+1})}{D^t(\mathbf{X}^t, \mathbf{Y}^t)} \times \frac{D^{t+1}(\mathbf{X}^{t+1}, \mathbf{Y}^{t+1})}{D^{t+1}(\mathbf{X}^t, \mathbf{Y}^t)} \right]^{\frac{1}{2}}$$

This index can be split into two parts: (see next slide)

Use DEAP 2.1 of Tim Coelli, freely downloadable for calculating Malmquist index

The change in technical efficiency is given by:

$$\frac{D^{t+1}(\mathbf{X}^{t+1}, \mathbf{Y}^{t+1})}{D^{t}(\mathbf{X}^{t}, \mathbf{Y}^{t})}$$

The technical change is given by:

$$\left[\frac{D^{t}(\boldsymbol{X}^{t+1}, \boldsymbol{Y}^{t+1})}{D^{t+1}(\boldsymbol{X}^{t+1}, \boldsymbol{Y}^{t+1})} \times \frac{D^{t}(\boldsymbol{X}^{t}, \boldsymbol{Y}^{t})}{D^{t+1}(\boldsymbol{X}^{t}, \boldsymbol{Y}^{t})} \right]^{\frac{1}{2}}$$

Scale efficiency

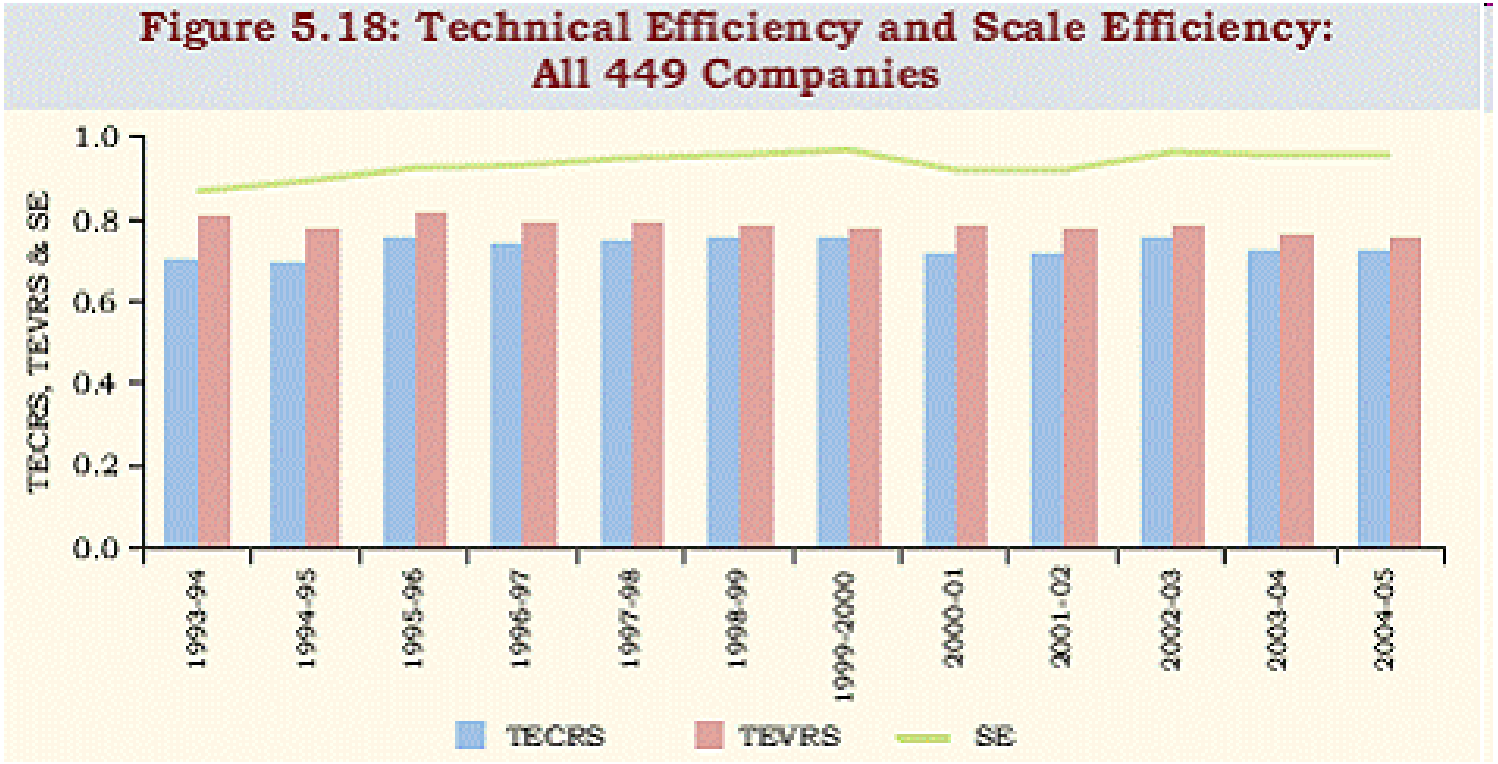
- Technical efficiency change can be split into pure technical efficiency change and scale efficiency change
  - Technical efficiency change index is obtained under CRS (constant returns to scale) assumption (TECI)
  - If VRS is allowed, one obtains pure technical efficiency change index (PTECI)
  - The ratio of TECI to PTECI is the scale efficiency change index

Computation of value of distance function

- This done by solving linear programming problems which use input output data (of the i’th firm and of all firms) as coefficients.

Pluses and minuses

- Advantages
- Can have multi inputs and multi outputs
- No functional form imposed,
- Hence technical change can be flexibly non-neutral
- No price information required
- But, there are disadvantages
- Data noise can be problematic
- When few firms and many dimensions (inputs and outputs), shadow prices can be “unusual”

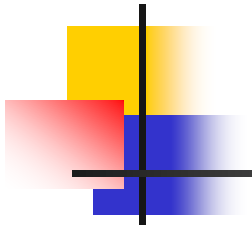


Productivity, Efficiency and Competitiveness of the Indian Manufacturing Sector, DRG study, Reserve Bank of India, June 2011

Industry	Average rate of TFP growth (%pa) based on Malmquist index (period 1994-95 to 2004-05)
Food & Beverages (60 firms)	-1.5
Chemical Industry (78 firms)	1.1
Metals and Metal Products (47 firms)	2.2
Machinery and Transport Equipment (MTE) (116 firms)	2.4
Textiles and Textile Products (53 firms)	0.5

Manufacturing (449 firms): 1.50 % p.a.

Tornqvist, ASI based, 1995-96 to 2003-04: 1.1% p.a.



Levinsohn-Petrin methodology

Key issues

- A number of studies have been undertaken on firm level productivity based on an estimated production function. The studies have used panel data.
- A methodological issue is: The input use decisions of a firm are likely to be related to the productivity changes and therefore the estimated parameters of the production function will be biased unless this aspect is taken into account in the method of estimation. The productivity estimates derived from the production function will also be biased.
- Certain approaches to tackle this problem have been suggested and applied.

Problems

- The simultaneity problem. The problem is that at least a part of the TFP will be observed by the firm at a point in time early enough so as to allow the firm to change the factor input decision. If that is the case, then profit maximization of the firm implies that the realization of the error term of the production function is expected to influence the choice of factor inputs. This means that the regressors and the error term are correlated, which makes OLS estimates biased.
- There is an additional problem caused by the fact that poor performing firm go out of the market and thus drop out of the sample.

Levinsohn-Petrin Approach

V= value added, L=labor, K=
capital, M=materials, E=energy, small letters for variable in log

Cobb-Douglas Production function

$$\begin{aligned} v_t &= \beta_0 + \beta_l l_t + \beta_k k_t + \omega_t + \eta_t \\ &= \beta_l l_t + \phi_t(k_t, m_t) + \eta_t \end{aligned}$$

Producti-vity shock

where

$$\phi_t(k_t, m_t) = \beta_0 + \beta_k k_t + \omega_t(k_t, m_t).$$

In a first stage, a third-order polynomial expansion in capital and materials is used to approximate $\phi(.)$ and then the coefficient of L is are estimated. In the second stage, the coefficient of K are estimated.

Levinsohn-Petrin Approach

- It is assumed that productivity shocks are reflected in materials used.
- One may alternatively use energy use as a proxy.

Available software for estimation: levpet.ado etc to be used in STATA; levpet.zip can be downloaded from the website of Amil Pertin; http://www.econ.umn.edu/~petrin/research/index.html

Computation of TFP Index

- Having obtained estimates of β_k and β_l the TFP of i'th firm in t'th year is obtained as

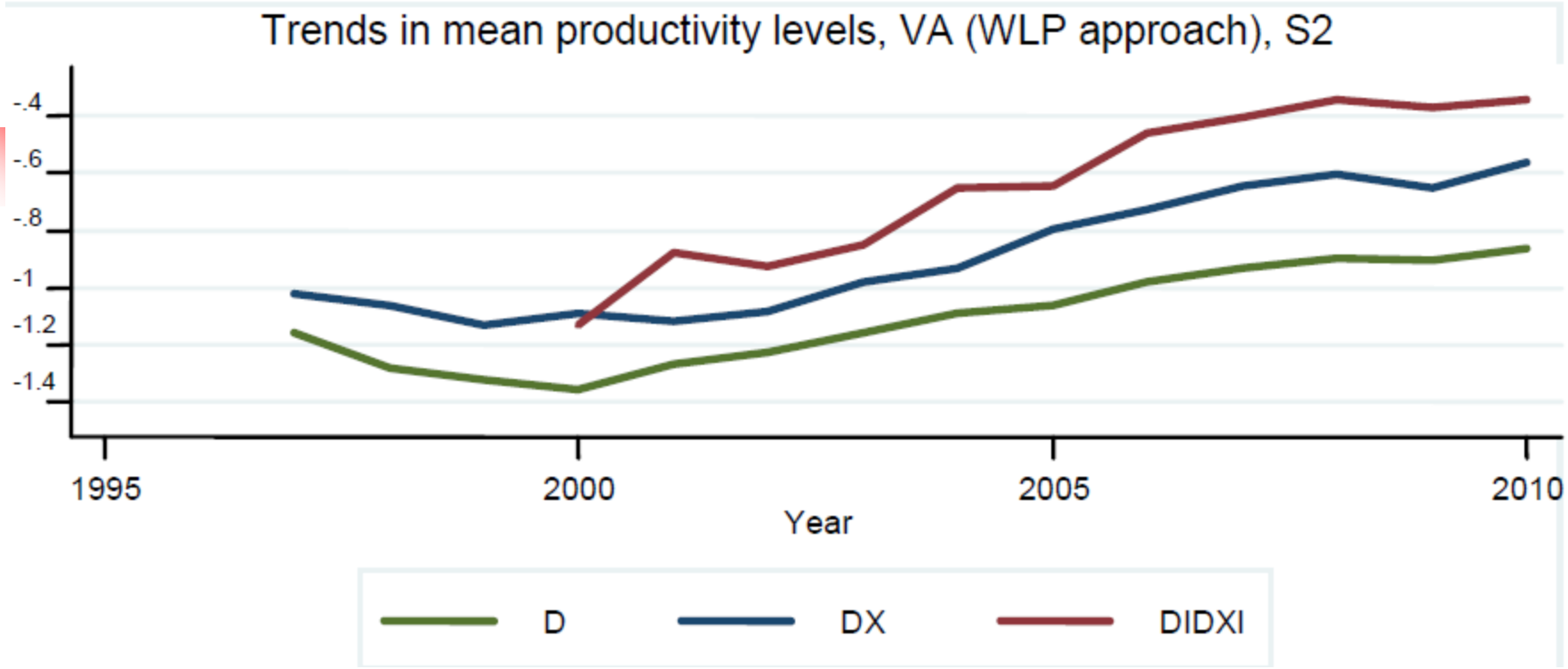
$$\ln TFP_{it} = \ln GVA_{it} - \hat{\beta}_k \ln capital_{it} - \hat{\beta}_l \ln labor_{it}$$

- This is compared with the TFP of a 'reference firm' to compute the index

$$\ln TFP_{it} - \ln TFP^r = \ln \frac{TFP_{it}}{TFP^r}$$

Sampat (2006) – estimates of TFP, Indian companies, 1994-2003

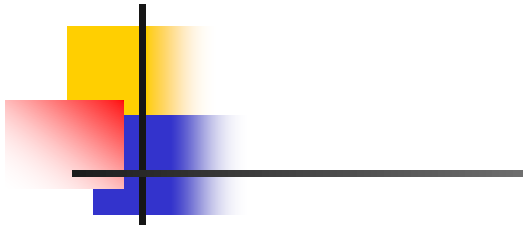
	TFP_OLS	TFP_LP	TFPG_OLS (% pa)	TFPG_LP (% pa)
Food & Bev	1.45	0.89	-1.4	-0.5
Textiles	0.98	0.68	2.1	2.2
Chemcal	1.31	1.32	-0.7	-0.7
Metals	1.26	1.08	-0.1	-0.4
Machnry	1.17	1.07	1.3	1.4



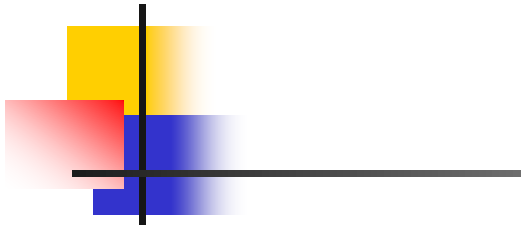
Foreign Involvement and Firm Productivity-An Analysis for Manufacturing Firms in India

Isha Chawla[1]

Paper to be presented at the 2015 conference of the Forum for Global Knowledge Sharing. See their website.



Thank You



Additional issues

Additional assumption for growth accounting using two-input framework

- It is assumed that the production function is separable – labour and capital are separable from the intermediate inputs.

Separability

- The use of value added function assumes that the production function is separable.
- $Q=f(L,K,M,E,S,t)=g(V(L,K,t), M, E, S)$
- Tests of separability often show that the assumption of separability is not justified.

Issues in input measurement - Quality

- Labor input – adjustment for quality (education, experience, etc)
- Capital input – changing composition of capital input (building vs. machinery) – ICT capital stock and its growing importance
- Very few studies in India have made adjustment for change in quality.

Growth in Labour and Capital input, US economy, 1995-2002

- Labour, growth rates (% p.a.)
  - Hours worked : 1.16
  - Labour quality: 0.33
  - Labour input : 1.50
- Capital, growth rates (% p.a.)
  - Capital stock : 2.66
  - Capital quality : 2.27
  - Capital input : 4.92

Source: Jorgenson, Ho and Stiroh, Information Technology and the American Growth Resurgence, 2005

Stochastic production frontier

- Assuming the production function to be Cobb-Douglas, the model may be written as:

$$\ln y_i = \beta_0 + \sum_k \beta_k \ln x_{ki} + v_i - u_i \qquad u_i \geq 0$$

- v is the two-sided “noise” component, and u is the one-sided, non-negative technical inefficiency component. Since the error term has two components, the model is referred to as composed error model.

Stochastic production frontier – normal and half normal case

- Production function is taken as:

$$\ln y_i = \beta_0 + \sum_k \beta_k \ln x_{ki} + v_i - u_i \quad u_i \geq 0$$

- The following additional assumptions are made:
  - v_i is iid $N(0,\sigma_v^2)$
  - u_i is iid $N^+(0,\sigma_u^2)$
  - v_i and u_i are distributed independently of each other and of the regressors (i.e. inputs)